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STRATEGIC INCENTIVES FOR KEPPING ONE SET OF BOOKS UNDER THE ARM'S LENGTH PRINCIPLE*

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Abstract

The OECD recommendation that transfer prices between parent firms and their subsidiaries be consistent with the Arm's Length Principle (ALP) for tax purposes does not restrict internal pricing policies. I show that under imperfect competition parents' accounting policies determine the properties of market outcomes: if parents keep one set of books (i.e., their internal transfer prices are consistent with the ALP), then competition in the external (home) market softens (intensifies) relative to the equilibrium where parents and subsidiaries are integrated. In contrast, if firms keep two sets of books (i.e., their internal transfer prices differ from those used for tax purposes) or maintain asymmetric accounting policies, then competition intensifies in both markets. Keeping one set of books is equilibrium in most of the parameter space.

Keywords: Transfer pricing regulation, Arm´s Length Principle, imperfect competition, vertical separation.

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1 Introduction

Policy makers have become increasingly aware of the possible use of transfer prices as a device for shifting profits into low tax jurisdictions. Transfer pricing policies have important implications since exports and imports from related parties are a dominant portion of trade flows – see Bernard, Jensen and Schott (2009). To moderate the incentives for firms to use transfer prices to shift profits from high to low tax jurisdictions for reasons unrelated to the economic nature of the transactions, most governments follow taxation policies that are based on the OECD Transfer Pricing Guidelines for Multinational Enterprises and Tax Administrations, which recommend that, for tax purposes, internal pricing policies be consistent with the Arm's Length Principle (ALP); i.e., that transfer prices between companies of multinational enterprises for tax purposes be established on a market value basis, thus comparable to transactions between independent (unrelated) parties -see [21].

Transfer prices serve both the purpose of allocating costs to different subsidiaries and determine tax liability of parents and subsidiaries. Aware of this problem, a growing number of multinational firms use an internal transfer price that differs from those used for tax purposes. This is a legal practice in the OECD countries. The only constraint is that transfer prices for tax purposes must be consistent with the ALP. Given that there is no statutory requirement, the incentive and tax transfer prices may differ. Therefore, an immediate question is whether firms separate their internal transfer prices from those used for tax purposes.

Using the terminology of Hyde and Choe (2005) and Dürr and Göx (2011), when firms use the same transfer price for tax reporting and for providing incentives, it is said that they keep *one set of books*, and when firms use different transfer prices for each purpose, it is said that they keep *two sets of books*.

In the absence of delegation, the choice between keeping one or two sets of books is not a matter. Thus, the strategic role of accounting policy is not driven by the oligopolistic market setting rather than by the decentralization of decision making. Under delegation, the choice between keeping one or two sets of books is relevant, even if tax rates are equal across jurisdictions.

Regarding theoretical studies on the optimal accounting strategy by decentralized firms which comply with tax rules, results are not conclusive. Specifically, these results depend on considering the presence of competition. Empirical evidence on the use of alternative accounting system is also mixed– see Dürr and Göx (2011) for a review of this literature.

First, abstracting from competition consideration, theoretical literature on this topic has established the superiority of keeping two sets of books whenever the tax and incentives objectives are conflicting -see Baldenius, Melumad and Reichelstein (2004).

Second, considering the possibility of competition, Göx (2000) and Dürr and Göx (2011) study the equilibrium accounting and transfer pricing policies in a multinational duopoly with price competition in the final product market. They find that the firms in a duopoly can benefit from strategically using the same transfer price for tax and managerial purposes instead of using separate transfer prices for both objectives. According to their results, firms in industries with a small number of competitors can benefit from using the same transfer price for tax and managerial purposes even if the tax and managerial objectives are conflicting. Therefore if firms keep one set of books, ALP may reinforces the effect of vertical separation in softening competition – see Vickers (1985), Fershtman and Judd (1987), Sklivas (1987), Alles and Datar (1998).

In this paper, taking compliance with the tax rules as given (i.e., transfer prices for tax purposes are consistent with the ALP); we study the optimal accounting strategy by decentralized multinational firms which compete in quantities in a context of imperfect competition.[1]

As Göx (2000), Arya and Mittendorf (2008) and Dürr and Göx (2011), we believe the accounting policy serves as commitment device since it is not often changed given the administrative and consulting costs associated with these changes.[2] Moreover,

[1]Quantity competition provides a reduced form model for the analysis of more complex forms of imperfect competition; e.g., capacity choice followed by some kind of price competition -see Kreps and Scheinkman (1983) and Moreno and Ubeda (2006).

[2]Other means of competitive commitment have been detailed in the literature, including distorting managerial compensation -Fershtman and Judd, (1987); Sklivas, (1987)-, sinking capacity investments -Dixit, (1980); Spence, (1977)-, building inventories -Ware, (1985)-, limiting information

accounting policies tend to be a matter of public record (in, for example, management discussion in annual reports, SEC filings and tax authority pricing agreements).

In our framework there are two markets, which we refer to as the Latin market (or home market) and the Greek market (or external market). There are two firms engaging in Cournot competition in the Latin market. These firms have subsidiaries, which in turn engage in Cournot competition in the Greek market. As customary, we assume that parents maximize consolidated profits, while subsidiaries maximize their own profits. Since competition in the Latin market provides a market price to impose on comparable market transactions, parents use this price to satisfy both cost and tax accounting requirements if keeping one set of books. If parents keep two sets of books, Latin market provides a market price only for tax purposes. Specifically, the analysis is based on a three stage non-cooperative game under complete information. Parents choose their accounting policy and then compete in quantities in a home market and set the prices at which they sell the good to their subsidiaries (either directly or indirectly via their output choices), which in turn compete in quantities an external market. (The decisions of the subsidiaries in the third stage are solely determined by the outcome of the second stage game.[3]) We show that parents' accounting policies determine the properties of market outcomes. Moreover, we obtain that collusion may be sustained in equilibrium. Before characterizing equilibria of this game, we analyze the properties of each subgames (i.e.; when both firms keep one set of books, when both firms keep two sets of books, as well when one firm keeps one set of books and the other keeps two sets of books).

In the subgame where both parents adopt one set of books (i.e., a parent must transfer the good to its subsidiary at the home market price), parent output decisions must internalize its impact on the transfer price of its subsidiary, and its subsidiary's rival. One set of books thus provides parents with an instrument to soften competition

acquisition -Einy et al., (2002); Gal-Or, (1988)-, and cost allocation rules -Gal-Or, (1993); Hughes and Kao, (1998).

[3]Since the outcome of the first stage game becomes known before the market stage of the game, the subsidiaries can infer the corresponding internal transfer price from their knowledge about the other firm's accounting policy and perfectly predict its internal transfer price even if it is not observable per se -see Alles and Datar (1998).

in the external market. Since a parent influences its transfer price via its output decision in the home market, competition is more aggressive in this market. Total profits under one set of books are above profits at the equilibrium where parents and subsidiaries are integrated. Hence using one set of books may provide a rationale for vertical separation. Implications of introducing taxes under one set of books are as follows. If tax rates are equal across jurisdictions, maximizing the gross or net process lead to the same result. If tax rates are different across jurisdictions, using one set of books also provides tax saving. In particular, when the external market offers a tax advantage over the home market, quantity in the home market is cut in order to increase the transfer price and therefore, every additional unit sold in the external market at a transfer price reduces the firm's tax bill. Nevertheless, in this setting what prevents a parent from decreasing its output further is that a cut in output also reduces its subsidiary's rival tax bill.

In the subgame where both parents adopt two sets of books (i.e., parent firms use internal transfer price that differs from that used for tax purposes), internal transfer prices open up the possibility to gain a Stackelberg advantage in the external market. Parents reduce their internal transfer prices below marginal cost in order to take advantage in the external market, creating a short of *prisoners' dilemma*. Implications of introducing taxes under two sets of books are as follows. If tax rates are equal across jurisdictions, maximizing the gross or net profit lead to a different result: a parent has an incentive to reduce the market price in the home market by increasing its output and at the same time reduces its internal transfer price, thus increasing its subsidiary's rival tax liability without affecting the marginal cost of its own subsidiary. Therefore, if both firms keep two sets of books together with a transfer pricing regulation consistent with the ALP competition intensifies in both markets relative to the equilibrium where parents and subsidiaries are integrated. Thus if tax rates are equal across jurisdictions, neither benefit from competition consideration nor tax bill savings exists when parents use two sets of books. Nevertheless if tax rates are different across jurisdictions, this accounting policy may reduce tax bill.

In the subgame with asymmetric accounting policies (i.e., one parent choosing one set of books and other parent choosing two sets of books), parent using two sets of books becomes the dominant producer in the external market, since its internal transfer price is lower than home market price, while parent using one set of books becomes the dominant producer in the home market because increasing its output in this market alleviates the double marginalization that arises in the external market. The total output (total profits) in both markets are above (below) the standard Cournot level. Nevertheless profits of the parent using two sets of books exceed this level.

Adding the first stage to the game, whereby parents choose their accounting policy, leads to a variety of equilibrium depending on market sizes and tax rates. Restricting attention to (pure strategy) subgame perfect equilibrium, the possible types of the game varies from a *prisoners' dilemma* (with a unique Pareto inefficient Nash equilibrium in which both parents choose two sets of books) to a *game of chicken* (with two pure strategy Nash equilibria, in these equilibria one firm uses one set of books and the other uses two sets of books) or a *coordination game* (with two pure strategy Nash equilibria, one in which both parents choose two sets of books, and another one in which choose one set of books). Also, parameter constellations of market sizes and tax rates can be found such the type of the game is a *cooperation game* (with a unique Pareto efficient Nash equilibrium in which both parents choose one set of books).

Parent's strategic behavior implies that keeping one set of books may be sustained as an equilibrium for most of the size difference between markets, when the tax rates are high. Moreover, this equilibrium is unique when the both markets are similar in size.

Our analysis contributes to the transfer pricing literature by broadening the understanding of the potential incentives for the choice of the accounting policy. A central premise in most related literature is that multinational firms set the same transfer price for tax and incentive purposes (i.e., keeping one set of books) –see Schjelderup and Sorgard (1997), Korn and Lengsfeld (2007), Nielsen et al. (2008) and Lemus and Moreno (2011). In these papers one set of books is taken as given and is not a matter of choice. Here we endogenize that choice and show that one set of books may be sustained as an equilibrium under broad conditions.

Since keeping one set of books provides parents with an instrument to soften competition in the external market, our analysis offers a convincing explanation of how the choice of the accounting policy can serve as a precommitment device. In our setting, regulatory constraint (i.e., transfer prices for tax purposes must be consistent with the ALP) commits the firms to the adoption of a particular accounting policy (i.e., one or two sets of books).[4]

In addition, our model contributes to the literature on strategic delegation by broadening the understanding of the potential benefits of decentralization, an organizational structure whose motivation is not well understood when firms compete in quantities. Dürr and Göx (2011) analyzed this question when firms compete in prices. Their results reinforce the effect of vertical separation in softening competition when firms keep one set of books. Nevertheless, novelty of their results is limited since analogous conclusions, when firms compete in prices, were found by Vickers (1985).

Our analysis does not only broaden the theoretical understanding but it also provides testable empirical predictions depending on the differences in the market size and tax rates.

The paper is organized as follows. Section 2 introduces the basic setup. Section 3 provides an equilibrium analysis under one set of books. Section 4 derives results for two sets of books. Section 5 studies the equilibrium with asymmetric accounting policies. Section 6 characterizes equilibria of this three stage non-cooperative game. Section 7 concludes.

[4]Arya and Mittendorf (2008) analyze market based transfer pricing as a strategic response in a similar setting. They show that ALP makes firms more aware of that excessive home market prices depress external production (i.e., the concern is about double marginalization) and may be more aggressive in the home market as a result. However, they do not recognize that ALP increases the prevailing transfer prices and thereby mitigate the prisoner's dilemma in transfer pricing to get an edge in downstream competition. In their model, parents rely on intracompany discounts to manage tensions between the home and the external markets. Intracompany discounts are set prior to the stage of competition in the home market and serve as a precommitment device. Nevertheless, this device is somewhat contrived since parents must credibly bind themselves to these discounts. In our setting, it is regulatory restriction (i.e., ALP) that serves to credibly convey to external parties that the related party price is above marginal cost (when one set of books is the accounting policy chosen by both parents).

2 Model and Preliminaries

A good is sold in two markets, which we refer to as the Latin market and the Greek market. The inverse demands in the Latin and Greek markets are $p^d(q) = \max\{0, 1 - bq\}$ and $\pi^d(\chi) = \max\{0, 1 - \beta\chi\}$, respectively, where b and β are positive real numbers. Assuming that demands are linear facilitates the analysis and makes it easier to interpret the results. We assume that maximum willingness to pay in each market is equal.[5] Differences in the slope of the demands (i.e., of the parameters b and β) capture the impact of differences in the market size – the demand is greater the smaller the slope. The parameter $s := b/\beta$ is a proxy for the size of Latin market relative to that of the Greek market.[6]

The taxable income in the Latin and Greek markets is determined by this tax τ and $\tau + \Delta$, respectively. The parameter Δ is the differential tax rates of the Greek relative to the Latin market. Tax rates are assumed to be less than 1, reflecting the idea that policy makers are unable or unwilling to tax multinational firms with a 100 per cent profit taxation.[7] When $\Delta > 0$ ($\Delta < 0$), the Latin (Greek) market is a tax heaven.

There are two firms producing the good at same constant marginal cost, which is assumed to be zero without loss of generality. Firms engage in Cournot competition in the Latin market, and have subsidiaries which in turn engage in Cournot competition in the Greek market.

We assume throughout that for tax purposes transfer prices must be consistent with the ALP; i.e., that the taxable income of a subsidiary that produces χ_i is $(\pi - p)\chi_i$, where π and p are the market prices in the Greek and Latin markets, respectively. Under this assumption the consolidated profits of firm i as a function of parents and

[5] Lemus and Moreno (2011) provide an equilibrium analysis when firms use one set of books, in which willingness to pay in each market are different.

[6] This assumption about willingness to pay holds if preferences over the good and/or range of income per capita are similar in the Latin market and in the Greek market. As regards market sizes, $\beta > b$ occurs if the number of people demanding the good in the Latin market is larger than in the Greek market.

[7] Dynamic allocative distortions associated with taxations (100 per cent profit taxation removes all incentive to do one thing rather than another) place constraints on profit taxation.

subsidiaries outputs is

$$\Pi_i(q_1,q_2,\chi_1,\chi_2) = \tag{1}$$
$$= (1-\tau)\,p^d(q_1+q_2)q_i + (1-\tau-\Delta)\,\pi^d\,(\chi_1+\chi_2)\,\chi_i + \Delta p^d(q_1+q_2)\chi_i.$$

We refer to the case where parents use the same transfer prices for internal and tax purposes as keeping *one set of books*. If a parent firm uses an internal transfer price that differs from that used for tax purposes, its subsidiary receives the good at a transfer price t_i (which is a non market based transfer prices) but the taxable incomes of the parent and subsidiary are determined by p. We refer to this case as keeping *two sets of books*.

Parent firms seek to maximize after tax consolidated profits, independently of whether they keep one or two sets of books; since the cost of production is zero, the consolidated profits are just the sum of the after tax revenues of the parent and the subsidiary. A subsidiary maximizes its own profit, which is the difference, after tax, between its revenue and its cost. A subsidiary' unit cost is just its transfer price. We identify a parent and its subsidiary firm with the same subindex $i \in \{1,2\}$.

In the absence of delegation, the choice between keeping one or two sets of books is not a matter. If parents do not delegate but rather compete in quantities also in the Greek market, the equilibrium outcome in both markets is independent of type of accounting.[8] In particular, if tax rates in both markets are identical, the equilibrium outcome is just the Cournot outcome in both markets.

In the Cournot equilibrium of a duopoly where the market demand is $P^d(Q) = \max\{0{,}1-BQ\}$, firms' constant marginal costs are $(c_1,c_2) \in R^2_+$ and the taxable income is determined by this tax τ, the market price P^C, the output Q_i^C and profit Π_i^C of firm i are

$$(P^C, Q_i^C, \Pi_i^C) = \left(\frac{1+c_1+c_2}{3}, \frac{1-2c_i+c_{3-i}}{3B}, \frac{(1-\tau)\,(1-2c_1+c_2)^2}{9B} \right). \tag{2}$$

If the market is monopolized by a single firm whose constant marginal cost is $c \in R_+$, then the market equilibrium price P^M, output Q^M, and the firm's profit Π^M

[8]Hyde and Choe (2005) observe this fact in a monopoly setting.

are

$$(P^M, Q^M, \Pi^M) = \left(\frac{1+c}{2}, \frac{1-c}{2B}, \frac{(1-\tau)(1-c)^2}{4B} \right). \tag{3}$$

Using these formulae (2), we readily calculate the Latin's market Cournot equilibrium price p^C, output $q_i^C = q^C$ and profit $\Pi_i^C = \Pi_L^C$ of firm i as

$$(p^C, q^C, \Pi_L^C) = \left(\frac{1}{3}, \frac{1}{3b}, \frac{1-\tau}{9b} \right). \tag{4}$$

Using the formulae (3), we obtain the monopoly equilibrium price, output, and the monopoly's profit in the Latin market as

$$(p^M, q^M, \Pi_L^M) = \left(\frac{1}{2}, \frac{1}{2b}, \frac{1-\tau}{4b} \right). \tag{5}$$

When aggregate output is q, the total surplus generated in the market is given by

$$S(q) = \left(1 - \frac{Bq}{2} \right) q. \tag{6}$$

In the Latin market, the surplus at the Cournot equilibrium, S_L^C, is therefore

$$S_L^C = \frac{4}{9b}, \tag{7}$$

and the surplus at monopoly equilibria, S_L^M, is

$$S_L^M = \frac{3}{8b}. \tag{8}$$

Replacing b with β yields formulas analogous for the Cournot and monopoly equilibria in the Greek market. (These formulas assume that firms' constant marginal cost of production is zero). We use the notation χ^C, π^C, Π_G^C, S_G^C, and χ^M, π^M, Π_G^M, S_G^M, for the values of output, price, profit and surplus at the Cournot duopoly equilibrium, and monopoly equilibrium of the market, respectively.

3 One Set of Books

In this section, we assume that parents use the market price in the Latin market as the transfer price per intrafirm transaction, i.e., parents keep only *one set of books* to satisfy both cost and tax accounting requirements. Of course, this internal pricing scheme is consist with the ALP. We identify the subgame perfect equilibria (SPE henceforth) of the game. In this setup, parents act as "leaders" anticipating the reactions of subsidiary firms.

Assuming that the price in the Latin market is $p \geq 0$, each subsidiary $i \in \{1, 2\}$ chooses its output χ_i to solve

$$\max_{\chi_i \in \mathbb{R}_+} (1 - \tau - \Delta) (\pi^d (\chi_1 + \chi_2) - p)\chi_i.$$

Here p is the constant marginal cost of the subsidiary firms.[9] Using the formulae (2), we calculate the equilibrium outputs and price for $p \geq 0$ as

$$\chi_1^* = \chi_2^* = \hat{\chi}(p) = \frac{1-p}{3\beta}.$$

(Note that in the game played by subsidiaries equilibrium is unique.) The equilibrium outcome depends only on p, but do not depend directly on the tax rate in the Greek market $(\tau + \Delta)$.

Therefore, the equilibrium price in the Greek market is

$$\pi^* = \pi^d (2\hat{\chi}(p)) = \frac{1+2p}{3}.$$

A SPE of the game is profile of actions for parents 1 and 2, (q_1^*, q_2^*), and a pair of functions describing the subsidiaries strategies $(f_1^* (q_1^*, q_2^*), f_2^* (q_1^*, q_2^*))$ such that parents maximize consolidated profits and subsidiaries maximize their own profits. Then in a SPE the subsidiaries strategies are $f_i^* (q_1^*, q_2^*) = \hat{\chi}_i(p^d(q_1^*, q_2^*))$ for $i \in \{1, 2\}$, and parents, anticipating that subsidiaries reactions are described by $(\hat{\chi}_1, \hat{\chi}_2)$, choose their actions in order to maximize consolidated profits (Π_i^O). Thus, Parent i chooses its output q_i in order to solve

$$\max_{q_i \in \mathbb{R}_+} \Pi_i^O(q_1, q_2),$$

where

$$\Pi_i^O(q_1, q_2) = \Pi_i(q_1, q_2, \hat{\chi}_1 \left(p^d(q_1 + q_2)\right), \hat{\chi}_2 \left(p^d(q_1 + q_2)\right)),$$

[9] Dürr and Göx (2011) assume that firms can arbitrarily choose a transfer price from an allowable exogenous range of ALP prices, withstanding a possible examination of authorities in the two markets.

continue to be the same formula as given by (1).

The first-order condition for profit maximizing is

$$\begin{aligned}\frac{\partial \Pi_i^O}{\partial q_i} &= (1-\tau)\left(\frac{dp^d}{dq}q_i + p^d\right) + (1-\tau)\frac{dp^d}{dq}\left(\frac{\partial \hat{\pi}}{\partial p}\hat{\chi}_i + \frac{\partial \hat{\chi}_i}{\partial p}\hat{\pi}\right) + \\ &+\Delta\frac{dp^d}{dq}\left(\hat{\chi}_i\left(1-\frac{\partial \hat{\pi}}{\partial p}\right) + \frac{\partial \hat{\chi}_i}{\partial p}(\hat{\pi}-p)\right) = 0. \end{aligned} \tag{9}$$

The expression in (9) comprises three different terms. In what follows, we refer to first term as Cournot marginal revenue, to second term as competition effect[10] and to the last term as tax effect[11]. Competition effect is a consequence of vertical separation (i.e., delegation).

In the absence of delegation and taxation, the optimal quantity in each market is found by equating Cournot marginal revenue with marginal cost (which in the model is zero). In particular, the equilibrium in both markets is just Cournot output.

The sign of competition effect depends on the price level in the Latin market and the sign of tax effect depends on the sign of Δ:

For $p^d > \frac{3}{4}p^C$, the influence that competition effect has on the marginal profit of parent i is positive from

$$(1-\tau)\frac{dp^d}{dq}\left(\frac{\partial \hat{\pi}}{\partial p}\hat{\chi}_i + \frac{\partial \hat{\chi}_i}{\partial p}\hat{\pi}\right) = (1-\tau)\frac{4s}{9}\left(p^d - \frac{3}{4}p^C\right),$$

in (9), so that the optimal quantity in this market, ceteris paribus, is above the Cournot output. Intuitively, this quantity increase is favorable because it reduces the Latin market price and therefore, alleviates the double marginalization problem. Nevertheless, double marginalization problem remains (i.e., $p^d > 0$). By charging transfer prices above marginal cost (zero in this model) both parents can commit their subsidiaries to behave as softer competitors on the final product market. In

[10] Since Latin market price are observable, a parent takes into account that it can influence this price via its output decision in the Latin market. Thus, firms can use Latin market price strategically to affect output decisions for the external market. In this setting, a high Latin market price can be used to reduce the competition in the external market.

[11] If tax rates differ among jurisdiction, firms want to shift profits into the low tax jurisdictions by use of distorted transfer prices.

this setting, a parent takes into account that it can influence its transfer price only via its output decision in the Latin market. Hence, a parent output decision must internalize its impact on the transfer price of its subsidiary, and its subsidiary's rival. Therefore one set of books provides parents with an instrument to soften competition in the external market.

For $\Delta > 0$ the influence that tax effect has on the marginal profit of parent i is negative from

$$\Delta \frac{dp^d}{dq}\left(\hat{\chi}_i\left(1-\frac{\partial\hat{\pi}}{\partial p}\right)+\frac{\partial\hat{\chi}_i}{\partial p}\left(\hat{\pi}-p\right)\right)=-\frac{2s}{9}\Delta\left(1-p^d\right),$$

in (9), so that the optimal output in this market, ceteris paribus, is lower than output in a setting without taxes (or with equal tax rates between markets). Intuitively, this quantity reduction is favorable because it increases the transfer price and every additional unit that is sold in the Greek market at a transfer price reduces the subsidiary's tax bill.

In this setting, what prevents a parent from decreasing its output further in order to increase Latin market price is that a decrease in output also reduces its subsidiary's rival tax bill. The opposite holds for $\Delta < 0$.

For $\Delta > 0$ the influence that tax effect has on the marginal profit of subsidiary i is also negative. Intuitively, increasing p, given that tax effect in the Latin market is negative, acts as a marginal cost increase for subsidiaries. The opposite holds for $\Delta < 0$.

Solving the system of equations formed by the first-order condition of parents 1 and 2, we obtain their outputs

$$q_1^* = q_2^* = \frac{(1-\tau)(3b+9\beta)}{b\left((1-\tau)(8b+27\beta)+4b\Delta\right)} := q^O. \tag{10}$$

The equilibrium price in the Latin market is

$$p^d(2q^O) = \frac{(1-\tau)(2b+9\beta)+4b\Delta}{(1-\tau)(8b+27\beta)+4b\Delta} := p^O.$$

Substituting the value of p in equations $\hat{\chi}_i(p)$ and $\hat{\pi}(p)$ above, we obtain the subsidiaries' outputs,

$$\chi_1^* = \chi_2^* = \hat{\chi}(p^O) = \frac{(1-\tau)(2b+6\beta)}{\beta\left((1-\tau)(8b+27\beta)+4b\Delta\right)} := \chi^O, \tag{11}$$

and the equilibrium price in the Greek market,

$$\pi^d(2\chi^O) = \frac{(1-\tau)(4b+15\beta)+4b\Delta}{(1-\tau)(8b+27\beta)+4b\Delta} := \pi^O.$$

Note that if the taxes differential was zero (i.e., $\Delta = 0$), this outcome would also be optimal in a setting without taxes and maximizing the gross or net profits leads to the same result. For $\Delta > 0$, the output in both markets decreases with Δ. The opposite effect applies to the equilibrium quantity for $\Delta < 0$. Since prices in the Latin market increase with Δ, parents save on tax payments by using one set of books.

In particular, if $\Delta = 0$ and using (4) we can rewrite the expression for firms' output in the Latin market (10) as

$$q^O = q^C + \frac{1}{3(8b+27\beta)}.$$

Likewise, using the equation (5) we can write the expression for firms' output in the Greek market (11) as

$$\chi^O = \frac{\chi^M}{2} - \frac{3}{4(8b+27\beta)}.$$

Thus, the output in the Latin market is above the Cournot output and the output in the Greek market is below the Cournot output. Note also that double marginalization imposed by ALP leads to an output in the Greek market that is below the monopoly output.

We have

$$\frac{\partial q^O}{\partial \beta} = -\frac{9}{(8b+27\beta)^2} < 0,$$

and

$$\frac{\partial \chi^O}{\partial b} = \frac{6}{(8b+27\beta)^2} > 0.$$

The output in the Latin (Greek) market decreases (increases) with β (b). It is worthwhile responding to an increase of the Greek market size (i.e., a smaller β) with an increase of the output in the Latin market, thus reducing the transfer price (in order to alleviate the double marginalization problem) and avoiding a large reduction of the sales of the subsidiary.

The equilibrium output in the Latin market also satisfies

$$\lim_{\beta\to 0} q^O = q^C + \frac{1}{24b} := q_0^O,$$

and

$$\lim_{\beta\to\infty} q^O = q^C.$$

Thus, as the size of the Greek market becomes large (i.e., β becomes small), the output in the Latin market is above the Cournot output. Parents incentive to increase their output in order to alleviate double marginalization remains as the size of the Greek market becomes arbitrarily large. Of course, as the size of the Greek market becomes arbitrarily small (i.e., β approaches infinity), parents tend to ignore the double marginalization problem (as the profits in this market become negligible), and focus on the impact on their output decision on the Latin market, and their output approaches the Cournot output.

The equilibrium output in the Greek market satisfies

$$\lim_{b\to\infty} \chi^O = \frac{\chi^M}{2},$$

and

$$\lim_{b\to 0} \chi^O = \chi^C - \frac{1}{9\beta} = \frac{\chi^M}{2} - \frac{1}{36\beta} := \chi_0^O.$$

Thus, as the size of the Latin market becomes arbitrarily small (i.e., b approaches infinity), the revenues in this market become negligible, and parents output decisions mainly serve the purpose of committing to high prices in the Greek market.

Interestingly, keeping one set of books (i.e., internal transfer prices are consistent with the ALP) allows parents to attain perfect cooperation (i.e., they are able to sustain the monopoly outcome) when b approaches infinity. In this case, ALP is merely an instrument to avoid competition in the Greek market. When the size of the Latin market becomes arbitrarily large (i.e., b approaches zero), however, revenues mainly come from the Latin market and therefore, parents tend to ignore the impact of double marginalization in the Greek market, producing the Cournot output in the Latin market. Double marginalization leads to an output in the Greek market that is below the monopoly output.

Let us study the total profit and total surplus under one set of books. Total profits can be calculated using (1) and (4) as

$$\Pi^O = \Pi_L^O + \Pi_G^O = \Pi_L^C + \Pi_G^C + \frac{2(1-\tau)}{9}\frac{4s^2+22s+27}{\beta(8s+27)^2}, \tag{12}$$

and the total surplus can be calculated using (6) and (7) as

$$S_L^O + S_G^O = S_L^C + S_G^C - \frac{2}{9}\frac{20s^2 + 155s + 297}{\beta\left(8s + 27\right)^2}.$$

We summarize these results in the following proposition.

Proposition 1. *If both firms use one set of books and $\Delta = 0$, then in a SPE:*

(1.1) The output in the Latin market q^O is above the Cournot outcome, and increases with the size of the Greek market β, i.e.,

$$q^O > q^C \text{ and } \frac{\partial q^O}{\partial \beta} < 0,$$

and the output in the Greek market χ^O is below the Cournot outcome, and decreases with the size of the Latin market b, i.e.,

$$\chi^O < \chi^C \text{ and } \frac{\partial \chi^O}{\partial b} > 0.$$

Further, as β becomes large q^O approaches q^C, and as β becomes small q^O approaches q_0^O, where $q_0^O > q^C$. And as b becomes large χ^O approaches $\chi^M/2$, and as b becomes small χ^O approaches $\chi_0^O < \chi^C$, where $\chi_0^O < \chi^M/2$.

(1.2) The total profits are above the total profits at the Cournot equilibrium.

(1.3) The total surplus is below the total surplus at the Cournot equilibrium.

Keeping one set of books provides parent firms with an instrument to limit aggressive competition in the Greek market, and may allow them to encourage an outcome near the monopoly outcome when the size of the Greek market relative to that of the Latin market is large.[12] Of course, since a parent influences its transfer price only via its output decision in the Latin market, competition in this market is more aggressive and the output is above the Cournot output. Nevertheless, total profits are above at the Cournot profit. Thus, this accounting policy may provide a rationale for vertical

[12] Choe and Matsushima (2012) examine the effect of ALP on dynamic competition in imperfectly competitive markets and show that the ALP renders tacit collusion more stable. They consider a vertically related market with two upstream firms which supply to their downstream affiliate and other unrelated buyers in the downstream market. In applying the ALP, they consider as comparable uncontrolled price, the price the upstream firms charges unrelated buyers. In our setting, it is competition in the home market provides a reliable measure of an arm's length result.

separation. However, total surplus is below the surplus at the Cournot equilibrium, which raises some questions about the use of the ALP as a guideline for regulating transfer prices.

4 Two Sets of Books

We consider next the case where each parent uses *two sets of books* together with a transfer pricing regulation consistent with the ALP. In this scenario, subsidiary i's taxable income is $(\pi^d(\chi_1+\chi_2)-p)\chi_i$, where p is the price in the Latin market, whereas its gross profit is $\left(\pi^d(\chi_1+\chi_2)-t_i\right)\chi_i$, where t_i is the internal transfer price that parent i uses to allocate costs. Parent i's consolidated net profit as a function of the outputs of parents and subsidiaries continue to be the same formula as given by (1), $\Pi_i(q_1,q_2,\chi_1,\chi_2)$. We identify the subgame perfect equilibria (SPE henceforth) of the game as follows.

Assuming that the price in the Latin market is $p\in R_+$ and internal transfer prices are $(t_1,t_2)\in R$, each subsidiary $i\in\{1,2\}$ chooses its output χ_i to solve

$$\max_{\chi_i\in\mathbb{R}_+}\ \left(\pi^d(\chi_1+\chi_2)-t_i\right)\chi_i-(\tau+\Delta)\left(\pi^d(\chi_1+\chi_2)-p\right)\chi_i.$$

Solving the system of equations formed by the first-order condition of subsidiaries 1 and 2, we calculate their equilibrium outputs as

$$\chi_1^*=\chi_2^*=\tilde{\chi}_1(p,t_1,t_2)=\tilde{\chi}_2(p,t_1,t_2)=\frac{1-\tau-\Delta+(\tau+\Delta)p-2t_i+t_{3-i}}{3\beta(1-\tau-\Delta)}.$$

(Note that in the game played by subsidiaries equilibrium is unique.) The outcome in the Greek market depends on p, t_i and tax rate in the Greek market (depends on tax rate in the Greek market even if tax rates in both markets are identical; i.e., $\Delta=0$).

Assuming that $\tilde{\chi}_1(p,t_1,t_2)+\tilde{\chi}_2(p,t_1,t_1)\leq\frac{1}{\beta}$, the market price is

$$\begin{aligned}\tilde{\pi}(p,t_1,t_2)&=\pi^d(\tilde{\chi}_1(p,t_1,t_2)+\tilde{\chi}_2(p,t_1,t_2))\\&=\frac{1-\tau-\Delta-2(\tau+\Delta)p+t_1+t_2}{3(1-\tau-\Delta)}.\end{aligned}$$

A SPE of the game is profile of actions for parents 1 and 2, $(q_1^*,q_2^*,t_1^*,t_2^*)$, and a pair of functions describing the subsidiaries strategies $\left(f_1^*(q_1^*,q_2^*,t_1^*,t_2^*),f_2^*(q_1^*,q_2^*,t_1^*,t_2^*)\right)$

such that parents maximize consolidated profits and subsidiaries maximize their own profits. Then in a SPE the subsidiaries strategies are

$$f_i^*\left(q_1^*, q_2^*, t_1^*, t_2^*\right) = \tilde{\chi}_i(p^d(q_1^*, q_2^*), t_1^*, t_2^*) \text{ for } i \in \{1, 2\},$$

and parents, anticipating that subsidiaries reactions are described by $(\tilde{\chi}_1, \tilde{\chi}_2)$, choose their actions in order to maximize consolidated profits (Π_i^T). Thus, Parent i chooses its output q_i and its internal transfer price t_i in order to solve

$$\max_{(t_i, q_i) \in \mathbb{R} \times \mathbb{R}_+} \Pi_i^T(q_1, q_2, t_1, t_2),$$

where

$$\Pi_i^T(q_1, q_2, t_1, t_2) = \Pi_i(q_1, q_2, \tilde{\chi}_1(p^d(q_1 + q_2), t_1, t_2), \tilde{\chi}_2(p^d(q_1 + q_2), t_1, t_2)).$$

Parent i's first-order conditions for profit maximization are

$$\frac{\partial \Pi_i^T}{\partial t_i} = (1 - \tau - \Delta)\left(\frac{\partial \tilde{\pi}}{\partial t_i}\tilde{\chi}_i + \frac{\partial \tilde{\chi}_i}{\partial t_i}\tilde{\pi}\right) + \Delta\left(\frac{\partial \tilde{\chi}_i}{\partial t_i}p\right) = 0, \tag{13}$$

and

$$\begin{aligned}\frac{\partial \Pi_i^T}{\partial q_i} &= (1 - \tau)\left(p + \frac{dp^d}{dq}q_i\right) + (1 - \tau)\frac{dp^d}{dq}\left(\frac{\partial \tilde{\pi}}{\partial p}\tilde{\chi}_i + \frac{\partial \tilde{\chi}_i}{\partial p}\tilde{\pi}\right) + \\ &\quad + \Delta\frac{dp^d}{dq}\left(\tilde{\chi}_i\left(1 - \frac{\partial \tilde{\pi}}{\partial p}\right) + \frac{\partial \tilde{\chi}_i}{\partial p}(\tilde{\pi} - p)\right) = 0. \end{aligned} \tag{14}$$

The expression in (13) comprises two different terms. Using the same terminology as before, we refer to the first term as the competition effect on the internal transfer price t_i[13] and to the second term as tax effect on the internal transfer price t_i.

The sign of competition effect on the internal transfer price depends on the output level in the Greek market and the sign of tax effect on the internal transfer price depends on the sign of Δ:

For $\tilde{\chi}_i < \frac{6}{5}\chi^C$ the influence that the competition effect has on the marginal profit of parent i is negative from

$$(1 - \tau - \Delta)\left(\frac{\partial \tilde{\pi}}{\partial t_i}\tilde{\chi}_i + \frac{\partial \tilde{\chi}_i}{\partial t_i}\tilde{\pi}\right) = \frac{5}{3}\left(\tilde{\chi}_i - \frac{6}{5}\chi^C\right),$$

[13]Parent can use the internal transfer prices strategically to impact output decisions for the external market. In this setting, a low transfer price can be used to expand own market share in the external market.

in (13), so that the internal transfer price t_i, ceteris paribus, is lower than the marginal cost (zero in this model).[14] Note t_i is the constant marginal cost of the subsidiary firm. Intuitively, the internal transfer price t_i is lower than the marginal cost in order to render each subsidiary into a low cost competitor that behaves aggressively by increasing its quantity. The transfer price that optimizes managerial incentives t_i (which is a non market transfer pricing) opens up the possibility to gain a Stackelberg advantage in the Greek market. By reducing its internal transfer price below marginal cost, parents attempt to gain a kind of Stackelberg leader status, creating a short of prisoners' dilemma situation. As a consequence of the competition effect, the equilibrium outcome in the Greek market is more efficient than the Cournot outcome. Therefore in the absence of taxation, delegating output decision to subsidiaries encourages parents to compete more aggressively in the Greek market, relative to a setting in which parents exercise direct control of the subsidiary's output.

For $\Delta > 0$ the influence that tax effect has on the marginal profit of parent i is also negative from

$$\Delta \left(\frac{\partial \tilde{\chi}_i}{\partial t_i} p \right) = -\Delta \frac{2p}{3\beta \left(1 - \tau - \Delta\right)},$$

in (13), so that the internal transfer price t_i, ceteris paribus, is lower if Latin market offers a tax advantage over the Greek market. Intuitively, this cost reduction is favorable because it offsets the increase in its subsidiary's taxable income that occurs by the competition effect. The opposite holds for $\Delta < 0$.

The expression in (14) comprises three different terms. Again using the above terminology, we refer to first term as Cournot marginal revenue, to the second term as the competition effect on the output q_i and to the third term as the tax effect on the output q_i.

The signs of competition effect and tax effect on the output q_i depend on the output level in the Greek market and on Δ, respectively:

For $\tilde{\chi}_i > \frac{3\chi^C}{4}$ the influence that the competition effect has on the marginal profit of parent i is positive from

[14] If internal transfer price was equal to the marginal cost, the outcome in the Greek market would be Cournot outcome.

$$(1-\tau)\frac{dp^d}{dq}\left(\frac{\partial\tilde{\pi}}{\partial p}\tilde{\chi}_i+\frac{\partial\tilde{\chi}_i}{\partial p}\tilde{\pi}\right)=(1-\tau)\frac{4b(\tau+\Delta)}{3(1-\tau-\Delta)}\left(\tilde{\chi}_i-\frac{3\chi^C}{4}\right),$$

in (14), so that the output in the Latin market, ceteris paribus, is above the output at the Cournot equilibrium. Intuitively, this quantity increase is favorable because it rises the tax liability of its subsidiary's rival without affecting the marginal cost of its own subsidiary. Each parent can offset exactly its own tax liability increase, reducing its internal transfer price.

For $\Delta > 0$ the influence that the tax effect has on the marginal profit of parent i is negative from

$$\begin{aligned}&\Delta\frac{dp^d}{dq}\left(\tilde{\chi}_i\left(1-\frac{\partial\tilde{\pi}}{\partial p}\right)+\frac{\partial\tilde{\chi}_i}{\partial p}(1-2\beta\tilde{\chi}_i-p)\right)\\ =\ &-\Delta\frac{s(1-p)(\tau+\Delta)+3b(1-\tau-\Delta)\tilde{\chi}_i}{3(1-\tau-\Delta)},\end{aligned}$$

in (14), so that the output in this market, ceteris paribus, is lower if the Latin market is a tax heaven. Intuitively, this quantity reduction is favorable because it increases the transfer price and therefore reduces the firm's tax bill. The opposite holds for $\Delta < 0$.

Solving the system of equations formed by the first-order conditions of parents 1 and 2 we obtain their outputs and the internal transfer prices. In the Latin market, parents' outputs are

$$q_1^*=q_2^*=q^C+\frac{1}{\beta}\frac{d(\tau,\Delta)}{\theta(\tau,\Delta,s)}:=q^T,$$

where

$$d(\tau,\Delta)=\tau(1-\tau)-\frac{\Delta}{3}(2(1-\tau-\Delta)+(1+\tau)),$$

and

$$\theta(\tau,\Delta,s)=15(1-\tau)^2-\Delta(15(1-\tau)-2s(\tau-\Delta)).$$

Assuming that $2q^T\leq\frac{1}{b}$, the market price is

$$p^d(2q^T)=p^C-2s\frac{d(\tau,\Delta)}{\theta(\tau,\Delta,s)}:=p^T.$$

Equilibrium internal transfer prices are $t_1^* = t_2^* := t^T$, where

$$t^T = -\frac{1}{5} - \frac{10\tau(1-\tau)(s\tau - 4(1-\tau))}{5\theta(\tau,\Delta,s)}$$
$$-\Delta\frac{2(s(\Delta - \tau(6-5\Delta)) + 5(1-\tau)(4\tau - (1-\tau-\Delta)))}{5\theta(\tau,\Delta,s)}.$$

Substituting the values p^T and t^T in equations above we obtain the subsidiaries' outputs

$$\chi_1^* = \chi_2^* = \tilde{\chi}_1(p^*, t_1^*, t_2^*) = \tilde{\chi}_2(p^*, t_1^*, t_2^*) = \frac{6}{5}\chi^C - \frac{\Delta}{\beta}\frac{\delta(\tau,\Delta,s)}{\theta(\tau,\Delta,s)} := \chi^T,$$

and market price in the Greek market,

$$\pi^d(2\chi^T) = \frac{3}{5}\pi^C + 2\Delta\frac{\delta(\tau,\Delta,s)}{\theta(\tau,\Delta,s)} := \pi^T,$$

where $\delta(\tau,\Delta,s) = \frac{4}{5}s(\tau-\Delta) - 2(1-\tau)$.

For $\Delta > 0$, the output in the Latin market decreases with Δ if $\tau < \frac{1}{2}$ (see Appendix 1 for a proof of this assertion). Thus the tax effect on the output q_i prevails over the competition effect. Tax incentives make a high price desirable and therefore, parents increase the market price in the home market by reducing their outputs. Since Latin market price increases with Δ, parents save on tax payments by using two sets of books.

Increased tax rates on the Greek market may have a pro-competitive effect in this market by encouraging lower internal transfer price. Thus the reduction in the internal transfer price may prevail over the increase of tax bill as a result of increased tax rates and prices in the Latin market. Whether or not output in the Greek market decreases with Δ depends on the size difference between markets and on the value of Δ. (see Appendix 2 for a proof of this assertion).

In particular assuming that $\Delta = 0$ and using again (4), we can rewrite the expression for firms' output in the Latin market as

$$q^T = \begin{cases} \frac{3}{2}q^C - \frac{1}{6b}\frac{\gamma(\tau)-s}{\gamma(\tau)} = q^C + \frac{1}{2\gamma(\tau)}\chi^C & \text{if } s < \gamma(\tau) \\ \frac{3}{2}q^C & \text{if } s \geq \gamma(\tau) \end{cases},$$

the output in the Greek market as

$$\chi^T = \frac{6}{5}\chi^C,$$

and the internal transfer prices as

$$t^T = -\frac{1}{5} - \frac{\tau}{3\gamma(\tau)}s + \frac{8}{15}\tau,$$

where $\gamma(\tau) = \frac{5}{2}\frac{(1-\tau)}{\tau}$ (the gray curve in Figure 2 is the graph of γ).

Thus, the outputs in the Latin market and the Greek market are above the output at the Cournot equilibrium. On the one hand, parents reduce their internal transfer prices below marginal cost in order to take advantage in the external market, creating a short of prisoners' dilemma. On the another hand, parents increase their output (i.e., reducing the market price in the home market) in order to increase their subsidiary's rival tax liability without affecting the marginal cost of their own subsidiaries.

We have

$$\frac{\partial q^T}{\partial \beta} = -\frac{1}{6\beta^2\gamma(\tau)} < 0,$$

and

$$\frac{\partial \chi^T}{\partial b} = 0,$$

Thus, the output in the Latin market decreases with β. Parents respond to an increase of the size of the Greek market (i.e., as β becomes small) with an increase of the output in the Latin market, thus reducing the Latin market price, in order to raise the tax liability of its rival's subsidiary without affecting the marginal cost of its own subsidiary. The output in the Greek market is independent of the size b.

We have

$$\frac{\partial q^T}{\partial \tau} = \frac{1}{15\beta(1-\tau)^2} > 0,$$

and

$$\frac{\partial \chi^T}{\partial \tau} = 0.$$

The output in the Latin market increases with τ. The higher tax rates are, the larger output in the Latin market is. Parents respond to an increase of tax with an increase of the output in the Latin market, thus reducing the Latin market price, in order to raise the tax liability of its rival's subsidiary without affecting the marginal cost of its own subsidiary. The output in the Greek market is independent of the size τ.

Let us study the total profit and total surplus under two sets of books.

Firms profits in the Latin and Greek markets can be calculated using (1) and (4) as

$$\Pi_L^T = \begin{cases} \Pi_L^C - \frac{\tau}{45\beta}\frac{s+\gamma(\tau)}{\gamma(\tau)} & \text{if } s < \gamma(\tau) \\ 0 & \text{if } s \geq \gamma(\tau) \end{cases},$$

and $\Pi_G^T = \frac{18}{25}\Pi_G^C$, respectively. Therefore, total profits are

$$\Pi^T = \Pi_L^T + \Pi_G^T = \Pi_L^C - \frac{\tau}{45\beta}\frac{s+\gamma(\tau)}{\gamma(\tau)} + \frac{18}{25}\Pi_G^C \text{ if } s < \gamma(\tau), \tag{15}$$

and

$$\Pi^T = \Pi_L^T + \Pi_G^T = \frac{18}{25}\Pi_G^C \text{ if } s \geq \gamma(\tau). \tag{16}$$

The surplus in the Latin and Greek markets can be calculated using (6) and (7) as

$$S_L^T = \begin{cases} \frac{9}{8}S_L^C - \frac{1}{18b}\left(\frac{\gamma(\tau)-s}{\gamma(\tau)}\right)^2 = S_L^C + \frac{1}{18\beta}\frac{2\gamma(\tau)-s}{\gamma(\tau)^2} & \text{if } s < \gamma(\tau) \\ \frac{9}{8}S_L^C & \text{if } s \geq \gamma(\tau) \end{cases},$$

and $S_G^T = \frac{27}{25}S_G^C$, respectively. Therefore, total surplus is

$$S^T = S_L^T + S_G^T = \frac{9}{8}S_L^C + \frac{27}{25}S_G^C - \frac{1}{18b}\left(\frac{\gamma(\tau)-s}{\gamma(\tau)}\right)^2 \text{ if } s < \gamma(\tau), \tag{17}$$

and

$$S^T = S_L^T + S_G^T = \frac{9}{8}S_L^C + \frac{27}{25}S_G^C \text{ if } s \geq \gamma(\tau). \tag{18}$$

We summarize these results in the following proposition.

Proposition 2. *If both firms use two sets of books and $\Delta = 0$, then in a SPE:*
(2.1) The output in the Greek market is

$$\chi^T = \frac{6}{5}\chi^C,$$

and the output in the Latin market is

$$q^T = \begin{cases} \frac{3}{2}q^C - \frac{1}{6b}\frac{\gamma(\tau)-s}{\gamma(\tau)} = q^C + \frac{1}{2\gamma(\tau)}\chi^C & \text{if } s < \gamma(\tau) \\ \frac{3}{2}q^C & \text{if } s \geq \gamma(\tau) \end{cases}.$$

Moreover, q^T increases with β and converges to the efficient outcome as β becomes large.

(2.2) The total profits are below total profits at the Cournot equilibrium.

(2.3) The total surplus is above the total surplus at the Cournot equilibrium.

In summary, keeping two sets of books adhering to the ALP generates a subtle link between markets that may intensify competition in both markets. On the one hand, each parent attempts to make the subsidiary a lower cost competitor, in order to gain a competitive advantage in the external market, by reducing its internal transfer price. On the another hand, each parent attempts to increase the tax liability of its subsidiary's rival, in order to gain a competitive advantage in the external market, by reducing Latin market price (i.e. increasing its production). Therefore, using two sets of books opens the possibility to gain a competitive advantage in the external market by reducing own costs and increasing rival's one.

In the absence of the ALP, parents have also an incentive to employ below cost transfer prices in order to compel their subsidiaries to be more aggressive in the external market. However incentives in the home market are unchanged and the equilibrium outcome is just the Cournot outcome -see Lemus and Moreno (2011). Therefore, if both firms keep *two sets of books* together with a transfer pricing regulation consistent with the *ALP* competition intensifies in the external market relative to the equilibrium where both firms using transfer prices for tax purposes not linked to the external market price.

5 Asymmetric Accounting Policies

In this section we consider the case in which parent firms use asymmetric accounting policies. We assume that parent 1 uses the market price in the Latin market as the transfer price per intrafirm transaction, i.e., it keeps only *one set of books* to satisfy both cost and tax accounting requirements, while parent 2 uses *two sets of books*. Subsidiaries observe the price in the Latin market and the internal transfer policy before competing in quantities. We identify the subgame perfect equilibria (SPE henceforth) of the game. In this set up, parents act as "leaders" anticipating the reactions of the subsidiary firms. We assume throughout this section that $\Delta = 0$ (i.e., equal tax rates between markets).

Assuming that the price in the Latin market is $p \in R_+$, subsidiary 1 chooses its output χ_1 to solve

$$\max_{\chi_1 \in \mathbb{R}_+} (1-\tau)(\pi^d(\chi_1+\chi_2)-p)\chi_1.$$

Subsidiary 2, knowing the internal transfer price used by its parent $t_2 \in R$ chooses its output χ_2 to solve

$$\max_{\chi_2 \in \mathbb{R}_+} \left(\pi^d(\chi_1+\chi_2)-t_2\right)\chi_2 - \tau(\pi^d(\chi_1+\chi_2)-p)\chi_2.$$

Thus, the reaction functions of subsidiaries 1 and 2 are

$$R_1^\chi(\chi_2, p) = \max\left(\frac{1-p}{2\beta} - \frac{1}{2}\chi_2, 0\right),$$

and

$$R_2^\chi(\chi_1, p, t_2) = \max\left(\frac{1-p}{2\beta} + \frac{p-t_2}{2\beta(1-\tau)} - \frac{1}{2}\chi_1, 0\right),$$

respectively.

An equilibrium of the Greek market is a profile of the subsidiaries' outputs $(\overline{\chi}_1(p,t_2), \overline{\chi}_2(p,t_2))$ satisfying the system of equations

$$\begin{aligned} \chi_1 &= R_1^\chi(\chi_2, p), \\ \chi_2 &= R_2^\chi(\chi_1, p, t_2). \end{aligned}$$

Solving this system we get

$$\overline{\chi}_1(p,t_2) = \begin{cases} 0 & \text{if } t_2 < p-(1-\tau)(1-p), \\ \frac{(1-\tau)(1-p)-(p-t_2)}{3\beta(1-\tau)} & \text{if } p-(1-\tau)(1-p) < t_2 < p + \frac{(1-\tau)(1-p)}{2}, \\ \frac{1-p}{2\beta} & \text{if } t_2 > p + \frac{(1-\tau)(1-p)}{2}, \end{cases} \tag{19}$$

and

$$\overline{\chi}_2(p,t_2) = \begin{cases} \frac{1-p}{2\beta} + \frac{p-t_2}{2\beta(1-\tau)} & \text{if } t_2 < p-(1-\tau)(1-p), \\ \frac{(1-\tau)(1-p)+2(p-t_2)}{3\beta(1-\tau)} & \text{if } p-(1-\tau)(1-p) < t_2 < p + \frac{(1-\tau)(1-p)}{2}, \\ 0 & \text{if } t_2 > p + \frac{(1-\tau)(1-p)}{2}. \end{cases} \tag{20}$$

Note that in the game played by subsidiaries equilibrium is unique.

A SPE of the game is profile of actions for parents 1 and 2, (q_1^*, q_2^*, t_2^*), and a pair of functions describing the subsidiaries strategies $(f_1^*(q_1, q_2, t_2), f_2^*(q_1, q_2, t_2))$ such that parents maximize consolidated profits and subsidiaries maximize their own profits. As discussed above, the subsidiaries game has a unique equilibrium. Then in a SPE the subsidiaries strategies are $f_i^*(q_1, q_2, t_2) = \overline{\chi}_i(p^d(q_1, q_2), t_2)$ for $i \in \{1, 2\}$, and parents, anticipating that subsidiaries reactions are described by $(\overline{\chi}_1, \overline{\chi}_2)$, choose their actions in order to maximize consolidated profits $\left(\bar{\Pi}_i\right)$. Thus, Parent 1 chooses q_1 to solve

$$\max_{q_1 \in \mathbb{R}_+} \bar{\Pi}_1(q_1, q_2, t_2),$$

where

$$\bar{\Pi}_1(q_1, q_2, t_2) = \Pi_1(q_1, q_2, \overline{\chi}_1\left(p^d(q_1 + q_2), t_2\right), \overline{\chi}_2\left(p^d(q_1 + q_2)\right), t_2).$$

Denote by $R_1^q(t_2, q_2)$ the reaction function of Parent 1, i.e., the solution to Parent 1's profit maximization problem.

Likewise, Parent 2 chooses its output q_2 and its internal transfer price t_2 in order to solve

$$\max_{(t_2, q_2) \in \mathbb{R} \times \mathbb{R}_+} \bar{\Pi}_2(q_1, q_2, t_2),$$

where

$$\bar{\Pi}_2(q_1, q_2, t_2) = \Pi_2(q_1, q_2, \overline{\chi}_1\left(p^d(q_1 + q_2), t_2\right), \overline{\chi}_2\left(p^d(q_1 + q_2)\right), t_2).$$

Denote by $(R_2^q(q_1), R_2^t(q_1))$ the reaction functions of Parent 2, i.e., the solution to Parent 2's profit maximization problem.

Hence in a SPE of the game the profile of parents' actions, (q_1^*, q_2^*, t_2^*), satisfy the system

$$\begin{aligned} q_1^* &= R_1^q(t_2^*, q_2^*), \\ q_2^* &= R_2^q(q_1^*), \\ t_2^* &= R_2^t(q_1^*). \end{aligned}$$

In an interior SPE, i.e., such that the outputs of parent and subsidiaries are positive, the subsidiaries outputs are

$$\chi_1^* = \overline{\chi}_1\left(p^d(q_1^* + q_2^*), t_2^*\right) = \frac{(1-\tau)\left(1 - p^d(q_1^* + q_2^*)\right) - (p^d(q_1^* + q_2^*) - t_2^*)}{3\beta(1-\tau)} > 0,$$

and

$$\chi_2^* = \overline{\chi}_2\left(p^d(q_1^* + q_2^*), t_2^*\right) = \frac{(1-\tau)\left(1 - p^d(q_1^* + q_2^*)\right) + 2(p^d(q_1^* + q_2^*) - t_2^*)}{3\beta\,(1-\tau)} > 0.$$

Using these formulae we can solve the system of equations formed by parents 1 and 2 reaction functions to obtain

$$\begin{aligned}
q_1^* &= \frac{(1-2\tau)\,s^2 + 2\,(5-4\tau)\,s + 12\,(1-\tau)}{2b\,((1-2\tau)\,s + 18\,(1-\tau))}, \\
q_2^* &= -\frac{(1-2\tau)\,s^2 + 2\,(5-4\tau)\,s - 12\,(1-\tau)}{2b\,((1-2\tau)\,s + 18\,(1-\tau))}, \\
t_2^* &= -\frac{(1-2\tau)\,((1-3\tau)\,s + 12\,(1-\tau))}{2\,((1-2\tau)\,s + 18\,(1-\tau))}.
\end{aligned}$$

We calculate the equilibrium price in the Latin market,

$$p^d(q_1^* + q_2^*) = \frac{(1-2\tau)\,s + 6\,(1-\tau)}{(1-2\tau)\,s + 18\,(1-\tau)}.$$

Substituting the values t_2^* and $p^d(q_1^* + q_2^*)$ in the equations for χ_1^* and χ_2^* above we obtain the subsidiaries' outputs,

$$\begin{aligned}
\chi_1^* &= -\frac{1}{2\beta}\frac{(1-2\tau)\,s}{(1-2\tau)\,s + 18\,(1-\tau)}, \\
\chi_2^* &= \frac{1}{\beta}\frac{(1-2\tau)\,s + 12\,(1-\tau)}{(1-2\tau)\,s + 18\,(1-\tau)}.
\end{aligned}$$

For tax rates $\tau \in [0, 1/2)$, the equation above yields $\chi_1^* < 0$, and therefore an interior SPE does not exist.

Proposition 3. *Assume that parent firms use asymmetric accounting policies and* $\Delta = 0$. *If* $\tau \in [0, 1/2)$, *then an interior SPE does not exist.*

Since in almost all countries tax rates are below one half, we turn to studying the (corner) SPE that arise for $\tau \in [0, 1/2)$.[15] Let us be given a SPE. Note that a SPE is identified by (q_1^*, q_2^*, t_2^*), since subsidiaries outputs are given by $(\chi_1^*, \chi_2^*) =$

[15] Auerbach et al (2008) present evidence on trends in corporation tax revenues and the industrial composition of revenues for the G7 countries (France, United Kingdom, Germany, Italy, Japan, United States and Canada) over the period 1979 to 2006. They show that statutory corporation tax rates have been falling across the G7 economies and provide some evidence of convergence to main rates between 30% to 40%.

$\left(\overline{\chi}_1\left(p^d(q_1^*+q_2^*),t_2^*\right),\overline{\chi}_2\left(p^d(q_1^*+q_2^*),t_2^*\right)\right)$ in the equations (19) and (20). We establish some properties of SPE.

Claim 1. *If* $\chi_2^*>0$, *then* $t_2^*<p^*$ *and* $t_2^*\in(-\frac{1}{2},\frac{1}{4})$.

Proof. Assume that $\chi_2^*>0$. If $\chi_1^*>0$, then the first-order condition for Parent 2's profit maximization yields

$$t_2^*=R_2^t\left(q_1^*,q_2^*\right)=-\frac{1-\tau+(1-5\tau)\,p^d(q_1^*+q_2^*)}{4}.$$

Since $p^*=p^d(q_1^*+q_2^*)\geq 0$, then

$$\begin{aligned}t_2^*-p^* &= -\frac{1-\tau+(1-5\tau)\,p^*}{4}-p^*\\ &= -\frac{1-\tau}{4}\left(1+5p^*\right)<0.\end{aligned}$$

Moreover, since t_2^* increases with τ and $p^*\in(0,1)$, then $t_2^*\in(-\frac{1}{2},\frac{1}{4})$.

If $\chi_1^*=0$, then the first-order condition for Parent 2's profit maximization yields

$$t_2^*=R_2^t\left(q_1^*,q_2^*\right)=-\tau p^d(q_1^*+q_2^*).$$

Since $p^*=p^d(q_1^*+q_2^*)\geq 0$, then

$$t_2^*-p^*=-\left(1-\tau\right)p^*<0.$$

Moreover, since $p^*\in(0,1)$ and $\tau\in[0,1/2)$, then $t_2^*\in(-\frac{1}{2}$,0). $\square$

Claim 2. *If* $q_2^*=0$, *then* $q_1^*>0$.

Proof. Assume $q_2^*=0$. If $t_2^*<p^d(q_1^*+q_2^*)-(1-\tau)\left(1-p^d(q_1^*+q_2^*)\right)$, then $\chi_1^*=0$ and the first-order condition for Parent 1's profit maximization yields

$$q_1^*=R_1^q\left(t_2^*,0\right)=q^M>0.$$

If $p^d(q_1^*+q_2^*)-(1-\tau)\left(1-p^d(q_1^*+q_2^*)\right)<t_2^*<p^d(q_1^*+q_2^*)+\frac{(1-\tau)\left(1-p^d(q_1^*+q_2^*)\right)}{2}$, then χ_1^*,$\chi_2^*>0$ and the first-order condition for Parent 1's profit maximization yields

$$q_1^*=R_1^q\left(t_2^*,0\right)=\frac{1}{2b}\frac{9\left(1-\tau\right)^2+\left(5\left(1-2\tau\right)+3\tau^2\right)s+\left(1+\tau\right)st_2^*}{n(\tau,s)},$$

where $n(\tau,s):=9\left(1-\tau\right)^2+\left(1-2\tau\right)\left(2-\tau\right)s$. Note that $n(\tau,s)>0$ on $[0,1/2)$.

Since $\tau < 1/2$ by assumption, and $t_2^* > -\dfrac{1}{2}$ by Claim 1, we have

$$\begin{aligned} q_1^* &> \frac{1}{2b}\frac{9\left(1-\tau\right)^2 + \left(5\left(1-2\tau\right)+3\tau^2\right)s + \left(1+\tau\right)s(-\frac{1}{2})}{n(\tau,s)} \\ &= \frac{1}{2b}\frac{9\left(1-\tau\right)^2 + \frac{3s}{2}\left(1-2\tau\right)\left(3-\tau\right)}{n(\tau,s)} > 0. \end{aligned}$$

Finally, if $t_2^* > p^d(q_1^*+q_2^*)+\frac{(1-\tau)\left(1-p^d(q_1^*+q_2^*)\right)}{2}$, then $\chi_2^* = 0$ and the first-order condition for Parent 1's profit maximization yields

$$q_1^* = R_1^q\left(t_2^*, 0\right) = \frac{1}{b}\frac{s+2}{s+4} > 0. \ \square$$

Claim 3. $q_1^* > 0$.

Proof. Assume by way of contradiction that $q_1^* = 0$. If $t_2^* < p^d(q_1^* + q_2^*) - (1-\tau)\left(1 - p^d(q_1^* + q_2^*)\right)$, then $\chi_1^* = 0$, and therefore

$$\chi_2^* = \frac{1 - p^d(q_1^* + q_2^*)}{2\beta} + \frac{p^d(q_1^* + q_2^*) - t_2^*}{2\beta\left(1-\tau\right)},$$

by equation (20). Since $q_2^* > 0$ by Claim 2, then the first-order conditions for Parent 2's profit maximization are

$$\begin{aligned} q_2^* &= \frac{1}{b}\frac{2\left(1-\tau\right)^2 + \tau^2 s}{\tau^2 s + 4\left(1-\tau\right)^2} - \frac{1}{\beta}\frac{\tau}{\tau^2 s + 4\left(1-\tau\right)^2}t_2^*, \\ t_2^* &= \tau\left(1 - bq_2^*\right). \end{aligned}$$

Solving this system of equations we get $(q_2^*, t_2^*, \chi_2^*) = (q^M, \frac{\tau}{2}, \chi^M)$. However,

$$q_1^* = R_1^q\left(\frac{\tau}{2}, q^M\right) = \frac{q^M}{2} > 0,$$

contradicting that $q_1^* = 0$.

If $p^d(q_1^* + q_2^*) - (1-\tau)\left(1 - p^d(q_1^* + q_2^*)\right) < t_2^* < p^d(q_1^* + q_2^*) + \frac{(1-\tau)\left(1-p^d(q_1^*+q_2^*)\right)}{2}$, then $\chi_1^*, \chi_2^* > 0$, and therefore

$$\begin{aligned} \chi_1^* &= \frac{(1-\tau)\left(1 - p^d(q_1^* + q_2^*)\right) - (p^d(q_1^* + q_2^*) - t_2^*)}{3\beta\left(1-\tau\right)}, \\ \chi_2^* &= \frac{(1-\tau)\left(1 - p^d(q_1^* + q_2^*)\right) + 2(p^d(q_1^* + q_2^*) - t_2^*)}{3\beta\left(1-\tau\right)}. \end{aligned}$$

by equations (19) and (20). Since $q_2^* > 0$ by Claim 2, then the first-order conditions for Parent 2's profit maximization are

$$q_2^* = \frac{1}{2b}\frac{s\left(5\left(1-2\tau\right)+3\tau^2-3\left(3-5\tau\right)\right)+9\left(1-\tau\right)^2}{g\left(\tau,s\right)}+\frac{1}{2\beta}\frac{1-5\tau}{g\left(\tau,s\right)}t_2^*,$$
$$t_2^* = -\frac{1-3\tau}{2}+\frac{b}{4}\left(1-5\tau\right)q_2^*,$$

where $g\left(\tau,s\right) := 9\left(1-\tau\right)^2-\left(1-2\tau\right)\left(1+\tau\right)s$.

Solving this system of equations we get

$$\left(q_2^*,t_2^*,\chi_1^*,\chi_2^*\right) = \left(\frac{2}{b}\frac{2-s}{8-s},\frac{\tau\left(7+s\right)-3}{8-s},-\frac{1+s}{\beta\left(8-s\right)},\frac{6}{\beta\left(8-s\right)}\right).$$

Hence either $\chi_1^* < 0$ or $\chi_2^* < 0$, and therefore such a profile cannot be an SPE.

If $t_2^* > p^d(q_1^*+q_2^*)+\frac{(1-\tau)\left(1-p^d(q_1^*+q_2^*)\right)}{2}$, then $\chi_2^* = 0$, and therefore

$$\chi_1^* = \frac{1-p^d(q_1^*+q_2^*)}{2\beta},$$

by equation (19). Since $q_2^* > 0$ by Claim 2, then the first-order condition for Parent 2's profit maximization yields

$$q_2^* = q^M.$$

However,

$$q_1^* = R_1^q\left(q^M\right) = \frac{1}{2b}\frac{s+2}{s+4} > 0,$$

contradicting that $q_1^* = 0$. $\square$

Claim 4. $\chi_1^* > 0$.

Proof. Assume by way of contradiction that $\chi_1^* = 0$. Then

$$\chi_2^* = \frac{1-p^d(q_1^*+q_2^*)}{2\beta}+\frac{p^d(q_1^*+q_2^*)-t_2^*}{2\beta\left(1-\tau\right)} > 0,$$

by equation (20). Since $q_1^* > 0$ by Claim 3, the first-order condition for Parent 1's profit maximization yields

$$q_1^* = \frac{1-bq_2^*}{2b},$$

and the first-order conditions for Parent 2's profit maximization yield the system

$$\begin{aligned} q_2^* &= \max\left(0,\frac{1}{b}\frac{2\left(1-\tau\right)^2+\tau^2 s}{\tau^2 s+4\left(1-\tau\right)^2}-\frac{1}{\beta}\frac{\tau}{\tau^2 s+4\left(1-\tau\right)^2}t_2^*-\frac{2\left(1-\tau\right)^2+\tau^2 s}{\tau^2 s+4\left(1-\tau\right)^2}q_1^*\right), \\ t_2^* &= \tau\left(1-b\left(q_1^*+q_2^*\right)\right). \end{aligned}$$

Solving this system of equations we get $(q_1^*, q_2^*, t_2^*) = \left(q^C, q^C, \frac{\tau}{3}\right)$. Substituting these values in equation (19) yields

$$\chi_1^* = \bar{\chi}_1 \left(p^d(2q^C), \frac{\tau}{3}\right) = \frac{1}{12\beta} > 0,$$

contradicting that $\chi_1^* = 0$. □

Claim 5. $\chi_2^* > 0$.

Proof. Assume by way of contradiction that $\chi_2^* = 0$. Then

$$\chi_1^* = \frac{1 - p^d(q_1^* + q_2^*)}{2\beta} > 0,$$

by equation (19). Since $q_1^* > 0$ by Claim 3, the first-order condition for profit maximization of parents 1 and 2 yield the system

$$\begin{aligned} q_1^* &= \frac{1}{b}\frac{s+2}{s+4} - \frac{s+2}{s+4}q_2^*, \\ q_2^* &= \max\left(0, \frac{1}{2b}\left(1 - bq_1^*\right)\right). \end{aligned}$$

Solving this system of equations we get

$$(q_1^*, q_2^*) = \left(\frac{s+2}{b\left(s+6\right)}, \frac{2}{b\left(s+6\right)}\right).$$

In a SPE, the level of output $q_2^* = 2/b\left(s+6\right) > 0$ must maximize Parent 2's profit taking as given $q_1^* = \frac{s+2}{b(s+6)}$ and the subsidiaries reactions $(\bar{\chi}_1, \bar{\chi}_2)$. Then q_2^* solves the system given by the first-order conditions for Parent 2's profit maximization

$$\begin{aligned} q_2^* &= \frac{1}{2b}\frac{2s\left(13\tau - 5\left(2-\tau^2\right)\right) + 36\left(1-\tau\right)^2 - s^2\left(1-\tau\right)\left(2-\tau\right)}{\left(s+6\right)g\left(\tau, s\right)} + \frac{1}{2\beta}\frac{1-5\tau}{g\left(\tau, s\right)}t_2^*, \\ t_2^* &= -\frac{1}{4}\frac{2\left(5-13\tau\right) + s\left(1-\tau\right)}{s+6} + \frac{b}{4}\left(1-5\tau\right)q_2^*. \end{aligned}$$

The solution to this system is

$$\hat{q}_2 = \frac{s\left(s+10\right) - 16}{b\left(s-8\right)\left(s+6\right)}.$$

For $s > 0$, $\hat{q}_2 \neq 2/b\left(s+6\right)$, which leads to a contradiction. Hence $\chi_2^* > 0$. □

With these results in hand, we can now identify the parameter values of τ and $s = b/\beta$ for which a pure strategy SPE exists, and identify the equilibrium outputs and profits. Define $l(\tau) := 3\,(1-\tau)\,/\,(2-\tau)$, and $h(\tau) := 12\,(1-\tau)\,/\,(1+\tau)$. The functions l and h are both decreasing, and $l(\tau) < h(\tau)$ on $[0,1]$ – in Figure 1 the thin (resp. thick) curve is the graph of l (resp. h). Also write $r(\tau,s) := (5-7\tau)\,s + 24\,(1-\tau)$. Note that $r(\tau,s) > 0$ on $[0,1/2)$.

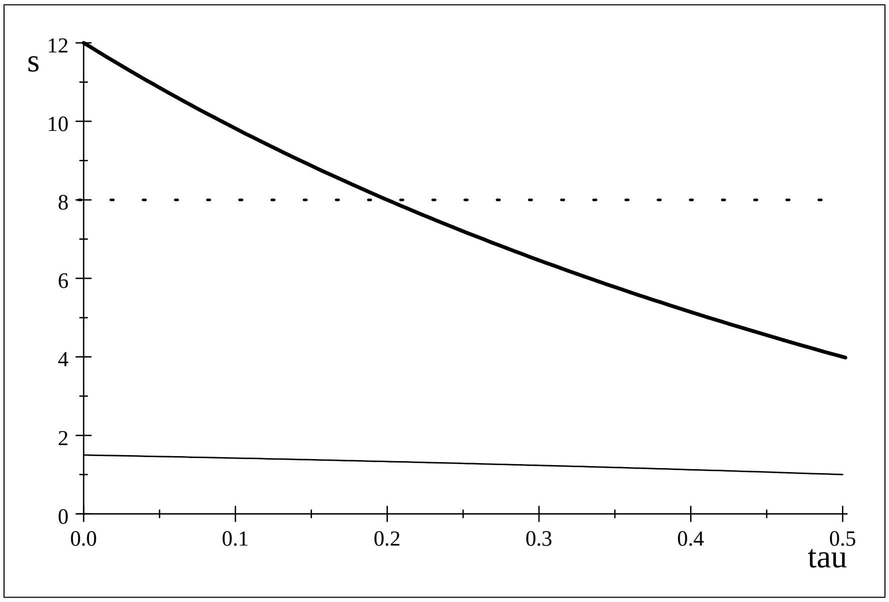

Figure 1: the Functions l and h.

Proposition 4: *Assume that $\tau < 1/2$ and $\Delta = 0$. If Parent 1 uses one set of books and Parent 2 uses two sets of books, then a unique (pure strategy) SPE exist whenever $l(\tau) < s < 8$, whereas no (pure strategy) SPE exists otherwise. Moreover:*

(4.1) If $l(\tau) < s < \min\{h(\tau), 8\}$, then the outputs of parents and subsidiaries in the unique SPE are

$$(q_1^*, q_2^*) = \left(2q^C + \frac{4\,(2-\tau)}{3b}\frac{s-l(\tau)}{r(\tau,s)}, 0\right),$$

and

$$(\chi_1^*, \chi_2^*) = \left(\frac{3}{4}\left(\chi^C - \frac{(1+\tau)}{\beta}\frac{h(\tau)-s}{r(\tau,s)}\right), \frac{3}{2}\left(\chi^C + \frac{(1+\tau)}{3\beta}\frac{h(\tau)-s}{r(\tau,s)}\right)\right),$$

and parents' profits are

$$\begin{aligned}
\Pi_1^* &= \frac{9}{16}\Pi_G^C - \frac{\left(1-\tau^2\right)\left(\left(7-17\tau\right)s^2-12\left(1-\tau\right)\left(s+16\right)\right)}{16b}\frac{h(\tau)-s}{r(\tau,s)^2}, \\
\Pi_2^* &= \frac{9}{8}\Pi_G^C + \frac{3\left(1-\tau^2\right)\left(\left(3-5\tau\right)s+20\left(1-\tau\right)\right)}{8\beta}\frac{h(\tau)-s}{r(\tau,s)^2}.
\end{aligned}$$

(4.2) If $h(\tau) \leq s < 8$, *then the outputs of parents and subsidiaries are*

$$\left(q_1^*, q_2^*, \chi_1^*, \chi_2^*\right) = \left(3q^C, 0, \frac{3}{4}\chi^C, \frac{3}{2}\chi^C\right),$$

and the parents' profits are

$$\left(\Pi_1^*, \Pi_2^*\right) = \left(\frac{9}{16}\Pi_G^C, \frac{9}{8}\Pi_G^C\right).$$

Proof. Since q_1^*,χ_1^*,$\chi_2^* > 0$, by claims 3, 4 and 5, and since by Proposition 3 there is no SPE such that these inequalities and $q_2^* > 0$ hold, then in a (pure strategy) SPE, when it exists, we have $q_2^* = 0$. Since χ_1^*,$\chi_2^* > 0$ by claims 4 and 5, then

$$\begin{aligned}
\chi_1^* &= \frac{\left(1-\tau\right)\left(1-p^d(q_1^*+q_2^*)\right)-\left(p^d(q_1^*+q_2^*)-t_2^*\right)}{3\beta\left(1-\tau\right)}, \\
\chi_2^* &= \frac{\left(1-\tau\right)\left(1-p^d(q_1^*+q_2^*)\right)+2\left(p^d(q_1^*+q_2^*)-t_2^*\right)}{3\beta\left(1-\tau\right)},
\end{aligned}$$

by equations (19) and (20). Since $q_1^* > 0$ by Claim 2 and $q_2^* = 0$, the first-order condition for Parent 1's profit maximization yields

$$q_1^* = R_1^q\left(t_2^*, 0\right) = \frac{1}{2b}\frac{s\left(5\left(1-2\tau\right)+3\tau^2\right)+9\left(1-\tau\right)^2}{n(\tau,s)} + \frac{1}{2\beta}\frac{1+\tau}{n(\tau,s)}t_2^*,$$

and the first-order condition for Parent 2's profit maximization yields

$$t_2^* = R_2^t\left(q_1^*, 0\right) = -\frac{1-3\tau}{2} + \frac{b}{4}\left(1-5\tau\right)q_1^*.$$

Solving this system of equations we get

$$\begin{aligned}
q_1^* &= \frac{1}{b} - \frac{\left(1+\tau\right)}{b}\frac{h(\tau)-s}{r(\tau,s)}, \\
t_2^* &= -\frac{1-\tau}{4} - \frac{1+\tau}{4}\frac{h(\tau)-s}{r(\tau,s)}\left(1-5\tau\right), \\
\chi_1^* &= \frac{\left(2-\tau\right)}{\beta}\frac{s-l(\tau)}{r(\tau,s)} \text{ and} \\
\chi_2^* &= \frac{2}{\beta}\frac{\left(1-2\tau\right)s+9\left(1-\tau\right)}{r(\tau,s)}.
\end{aligned}$$

(These values for q_1^*, χ_1^* and χ_2^* can be readily rewritten using the formulae given in (4.1) of Proposition 4.) Thus, $\chi_1^* \leq 0$ whenever $s \leq l(\tau)$. Since in equilibrium $\chi_1^* > 0$ by Claim 5, then a SPE does not exist when $s \leq l(\tau)$. Assume that $l(\tau) < s$. The equilibrium prices in the Latin is

$$p^* = p^d(q_1^*) = \frac{h(\tau) - s}{(1+\tau)\, r(\tau, s)}.$$

Thus, in order for $p^* > 0$ we must have $s < h(\tau)$. Assume that $h(\tau) > s$. The equilibrium price in the Greek markets is

$$\pi^* = \pi^d(\chi_1^* + \chi_2^*) = \frac{(1-2\tau)\, s + 9\,(1-\tau)}{r(\tau, s)} > 0.$$

In order to verify that the profile identified is SPE we need to show that the level of output $q_2^* = 0$ maximizes Parent 2's profits taking q_1^* as given. The system given by the first-order conditions for Parent 2's profit maximization is

$$q_2^* = \frac{1}{2b} \frac{3\,(1-\tau)\left(36\,(1-\tau)^2 - s\,(27 - \tau\,(32 + 11\tau)) + s^2\tau\left(\frac{29}{3} - \tau\right)\right) - 8s^2}{g\,(\tau, s)\, r(\tau, s)} + \frac{1}{2\beta} \frac{1 - 5\tau}{g\,(\tau, s)} t_2^*,$$
$$t_2^* = -\frac{s\,(1 - \tau\,(2 - 3\tau)) + 3\,(3 - 7\tau)\,(1-\tau)}{r(\tau, s)} + \frac{b}{4}\,(1 - 5\tau)\, q_2^*.$$

Solving this system we get

$$\bar{q}_2 = \frac{1}{b} \frac{4s\,(s+10) - 8s\tau\,(s+4) - 48\,(1-\tau)}{(s-8)\, r(\tau, s)}.$$

In order for $\bar{q}_2 \leq 0$ we must have

$$\gamma(\tau) := \frac{\sqrt{37 - 4\tau\,(19 - 10\tau)} - 5 + 4\tau}{1 - 2\tau} \leq s \leq 8.$$

Since $s > l(\tau) > \gamma(\tau)$ on $[0, 1/2)$, for $\bar{q}_2 \leq 0$ we must have $s < 8$. In summary, the profile of parents and subsidiaries outputs as well as the transfer price of parent 2 identified above forms a SPE when $l(\tau) < s < h(\tau)$ and $s < 8$, i.e., $l(\tau) < s < \min\{h(\tau), 8\}$. Thus, when this is the case there is a unique SPE and it is given by the formulae given in (4.1) of Proposition 4.

Now suppose that $s \geq h(\tau)$. Then in equilibrium $p^* = 0$, and therefore $q_1^* \geq \frac{1}{b}$. Then the first-order condition for Parent 2's profit maximization yields

$$t_2^* = R_2^t\left(\frac{1}{b}, 0\right) = -\frac{1-\tau}{4},$$

and therefore,

$$(\chi_1^*, \chi_2^*) = (\frac{3}{4}\chi^C, \frac{3}{2}\chi^C),$$

by equations (19) and (20). The equilibrium price in the Greek markets is

$$\pi^* = \pi^d(\chi_1^* + \chi_2^*) = \frac{3}{4}\pi^C.$$

In order for $q_2^* = 0$ to maximize the profits of Parent 2 taking as given $q_1^* = \frac{1}{b}$, the solution to the system defined the first-order conditions,

$$\begin{aligned} q_2^* &= -\frac{1-\tau}{2\beta}\frac{2-\tau}{g(\tau, s)} + \frac{1}{2\beta}\frac{1-5\tau}{g(\tau, s)}t_2^*, \\ t_2^* &= -\frac{1-\tau}{4} + \frac{b}{4}(1-5\tau)\, q_2^*. \end{aligned}$$

Solving this system of equations we get

$$\tilde{q}_2 = \frac{1}{\beta}\frac{1}{s-8}.$$

For $\tilde{q}_2 \leq 0$ we must also have $s < 8$. Hence the profile of outputs and transfer price define above forms a SPE when $h(\tau) < s < 8$.

Finally, if $s \leq l(\tau)$, then $\chi_1^* \leq 0$, and since in equilibrium $\chi_1^* > 0$ by Claim 5, then a SPE does not exist. And if $s \geq 8$, then whether $s < h(\tau)$, or $s \geq h(\tau)$ neither of the two candidate equilibria identified are SPE, and therefore a pure strategy SPE does not exist either.

The parents equilibrium profits for the cases $l(\tau) < s < \min\{h(\tau), 8\}$ and $h(\tau) < s < 8$ are readily obtained simply by substituting parents and subsidiaries outputs in the formulae of the consolidated profits. □

In an equilibrium in which parents use asymmetric accounting policies, the parent that uses one set of books, say Parent 1, has an incentive to increase its output in order to alleviate double marginalization (i.e., to decrease the cost of its subsidiary), whereas the parent that uses two sets of books, Parent 2, decreases its output all the way to zero in order to increase the cost of its subsidiary's rival. Thus Parent 1 becomes the dominant producer in the home market. Since $t_2^* < p^*$ by claims 1 and 5, Subsidiary 2 becomes the dominant producer in the external market. Equilibrium profits of Parent 2 uses two sets of books dominate equilibrium profits of Parent 1 uses one set of books (i.e., $\Pi_2^* > \Pi_1^*$, see Appendix 3).

Assume that $l(\tau) < s < \min\{h(\tau), 8\}$. Then the total output in the Latin market satisfies

$$q_1^* + q_2^* = q_1^* = 2q^C + \frac{4\,(2-\tau)}{3b}\frac{s - l(\tau)}{r(\tau,s)} > 2q^C,$$

and the total output in the Greek market satisfies

$$\chi_1^* + \chi_2^* = 2\chi^C + \frac{2-\tau}{3\beta}\frac{s - l(\tau)}{r(\tau,s)} > 2\chi^C.$$

Hence the surplus in both markets is above the surplus at the Cournot equilibrium, i.e., $S_L^* > S_L^C$ and $S_G^* > S_G^C$. Since $S_L^C > S_L^O$ and $S_G^C > S_G^O$ by Proposition 1, the surplus in both markets is above under one set of books. In the Appendix 4 we show that $S_L^* + S_G^* < S^T$ and therefore, the total surplus is below under two sets of books.

We have

$$\frac{\partial q_1^*}{\partial \beta} = -\frac{12\,(1-\tau)}{\beta^2 r(\tau,s)^2}\,(7-5\tau) < 0,$$

and

$$\frac{\partial\,(\chi_1^* + \chi_2^*)}{\partial b} = \frac{3\,(1-\tau)}{\beta^2 r(\tau,s)^2}\,(7-5\tau) > 0.$$

Thus, the output in the Latin (Greek) market decreases (increases) with β (b). Parent 1 responds to an increase of the size of the Greek market (i.e., a smaller value of β) with an increase of the output in the Latin market, thus reducing the Latin market price (in order to alleviate the double marginalization problem) and avoiding a large reduction of the sales of its subsidiary. The market share of subsidiary 2 increases with the size of the Latin since its output decreases with b. Subsidiary 1 is more (less) aggressive competitor in the Greek market as the profits in Latin market become negligible (large).

Also we have

$$\frac{\partial q_1^*}{\partial \tau} = 12\frac{s+2}{\beta r(\tau,s)^2} > 0,$$

and

$$\frac{\partial\,(\chi_1^* + \chi_2^*)}{\partial \tau} = 3s\frac{s+2}{\beta r(\tau,s)^2} > 0.$$

The output in the Latin market of parent 1 increases with τ. The higher tax rates are, the larger output in the Latin market of parent 1 is. This occurs because a larger Latin market output (to compensate for $q_2^* = 0$) tends to reduce the difference between the tax bill paid at the Latin and the Greek markets. The output of subsidiary 1 (2) increases (decreases) with τ. Parent 1 responds to an increase of tax with an increase of the output in the Latin market, thus reducing the Latin market price. A decrease in the Latin market price encourages the subsidiary 1 to behave more aggressively by expanding its output in the Greek market and thus causes subsidiary 2 to become less aggressive by reducing its outcome.

If $h(\tau) < s < 8$, then

$$q_1^* + q_2^* = q_1^* = 3q^C > 2q^C,$$

and

$$\chi_1^* + \chi_2^* = \frac{9}{4}\chi^C > 2\chi^C.$$

Hence the surplus in both markets is above the surplus at the Cournot equilibrium, i.e., $S_L^* > S_L^C$ and $S_G^* > S_G^C$. Since $S_L^C > S_L^O$ and $S_G^C > S_G^O$ by Proposition 1, the surplus in both markets is above under one set of books. In the Appendix 4 we show that $S_L^* + S_G^* < S^T$ and therefore, the total surplus is below under two sets of books.

Of course, our results would be symmetric if Parent 1 uses two sets of books and Parent 2 uses one set of books. Henceforth we use the superscripts $\bar{O}T$ and $O\bar{T}$ to refer to the outputs and profits of the firm using one and two sets of books, respectively, in a situation where parents use asymmetric accounting policies; i.e., $q^{\bar{O}T} = q_1^*$, $\chi^{\bar{O}T} = \chi_1^*$ and $\Pi^{O\bar{T}} = \Pi_1^*$, whereas $q^{O\bar{T}} = q_2^*$, $\chi^{O\bar{T}} = \chi_2^*$ and $\Pi^{O\bar{T}} = \Pi_2^*$, where the star values are those given in Proposition 4 above.

6 Endogenizing the Choice of Accounting Policies

We now turn to study parents choice of accounting policies. We assume that parents can commit to keeping either *one set of books* or *two sets of books.* This assumption is reasonable if, for example, the costs associated with changing the accounting policy are prohibitively high.[16] By choosing to keep one set of books, a parent commits to using the Latin market price as the transfer price per intrafirm transaction, regardless of its competitor actions. Likewise, by choosing to keep two sets of books, a parent

[16]Göx (2000) notes that a new accounting policy usually requires substantial investments in developing or acquiring software and in training employees and/or hiring consultants.

commits to using an internal transfer price to allocate costs, whatever action of its competitor.

In section 3, we identified the parents' profits when both parents choose one set of books, Π^O and in section 4, we identified the parents' profits when both parents choose two sets of books, Π^T. Likewise, in section 5 we identified the profits in a (pure strategy) SPE when parents choose asymmetric accounting policies, $\Pi^{\bar{O}T}$ and $\Pi^{O\bar{T}}$, where the superscripts $\bar{O}T$ and $O\bar{T}$ refer to the parent using one and two sets of books, respectively. Thus, at the stage of choosing their accounting policies parents, assuming that following their decisions a (pure strategy) SPE follows, parents' payoffs are described by the following matrix:

	O_2	T_2
O_1	Π^O, Π^O	$\Pi^{\bar{O}T}, \Pi^{O\bar{T}}$
T_1	$\Pi^{O\bar{T}}, \Pi^{\bar{O}T}$	Π^T, Π^T

Table 1: Parents' Choice of Accounting Policies.

We study the equilibria of this game. Recall from proposition 2 the function defining Π^T differ in the space (τ, s) depending on the sign of the inequality $s \gtreqless \gamma(\tau)$. Likewise, from proposition 4 the functions defining $\Pi^{\bar{O}T}$ and $\Pi^{O\bar{T}}$ differ in the space (τ, s) depending on the sign of the inequality $s \gtreqless h(\tau)$. In Appendix 5 we study the sing of $\Pi^T - \Pi^{\bar{O}T}$, which is the profit gain or loss to a parent that *deviates* to choosing *one set of books* from a situation where both parents choose *two sets of books.* In Appendix 6 we study the sign of $\Pi^O - \Pi^{O\bar{T}}$, which is the profit gain or loss to a parent that *deviates* to choosing *two sets of books* from a situation where both parents choose *one set of books.*

On the parameter space (τ, s) parents' profits configure the game described in Table 1 as a *prisoners' dilemma* (with a unique Pareto inefficient Nash equilibrium in which both parents choose two sets of books), a *game of chicken* (with one parent choosing one set of books and other parent choosing two sets of books), a *coordination game* (in which both parents choose two sets of books or both parents choose one set of books), or even to a *cooperation game* with a unique Pareto efficient Nash equilibrium (in which both parents choose one set of books).

In Appendixes 3 and 4 we show that $\Pi^T > \Pi^{\bar{O}T}$ and $\Pi^O > \Pi^{O\bar{T}}$ whenever $\min\{\gamma(\tau), h(\tau)\} < s < 8$. In this region, characterized by relatively high tax rates ($\tau > \frac{1}{5}$) and a large value of the size of the Greek market relative to that of the Latin ($s > \frac{5}{2}$), the game in Table 1 is a Coordination Game *(CO)* that has two pure strategy Nash equilibria, one in which both parents choose two sets of books, and another one in which choose one set of books.

When $s < \min\{\gamma(\tau), h(\tau)\}$ identifying the signs of $\Pi^T - \Pi^{\bar{O}T}$ and $\Pi^O - \Pi^{O\bar{T}}$ is cumbersome. We show that if $l(\tau) < s < 1.385$, then $\Pi^T < \Pi^{\bar{O}T}$. If $1.385 < s < \min\{\gamma(\tau), h(\tau)\}$ and the tax rates are not too high, then $\Pi^T > \Pi^{\bar{O}T}$ (see Figure 3 in Appendix 5). We also show that if $1.23 < s < 2.26$, then $\Pi^O < \Pi^{O\bar{T}}$. If $l(\tau) < s < 1.23$ or $2.26 < s < \min\{\gamma(\tau), h(\tau)\}$, there is a critical value $\widetilde{\tau}$ such that $\Pi^O \lesseqgtr \Pi^{O\bar{T}}$ whenever $\tau \lesseqgtr \widetilde{\tau}$ (see Figure 4 in Appendix 6). These results allow to identify the possible types of the game that Table 1 may give rise depending of the values of s and τ:

(i) If $l(\tau) < s < 1.23$, then $\Pi^T < \Pi^{\bar{O}T}$, and Table 1 describes either a Game of Chicken *(CH)* (when $\Pi^O < \Pi^{O\bar{T}}$) or a Cooperation Game *(CP)* (when $\Pi^O > \Pi^{O\bar{T}}$) depending on whether the value of τ is high or very high, respectively. In a *CH* game there are two pure strategy Nash equilibria, in these equilibria one firm uses one set of books and the other uses two sets of books. In a *CP* game it is a dominant strategy for both firms to use one set of books.

(ii) If $1.23 < s < 1.385$, then $\Pi^O < \Pi^{O\bar{T}}$ and $\Pi^T < \Pi^{\bar{O}T}$, and hence Table 1 describes a *CH* game.

(iii) If $1.385 < s < 2.26$, then $\Pi^O < \Pi^{O\bar{T}}$, and the game in Table 1 is either a Prisoners' Dilemma game *(PD)* (when $\Pi^T > \Pi^{\bar{O}T}$) or a *CH* game (when $\Pi^T < \Pi^{\bar{O}T}$), depending on whether τ is low or high, respectively. In a *PD* game keeping two sets of books is the unique equilibrium (and is in dominant strategies).

(iv) If $2.26 < s < \min\{\gamma(\tau), h(\tau)\}$, then there are parameter constellations such that $\Pi^T \lessgtr \Pi^{\bar{O}T}$ and/or $\Pi^O \lessgtr \Pi^{O\bar{T}}$. In this case, all four types of games (*PD*, *CO*, *CH* and *CP*) may emerge as the tax rate τ increases from low, to intermediate, to high values.

In Figure 2 below, the gray curve is the graph of the function γ, the thin curve

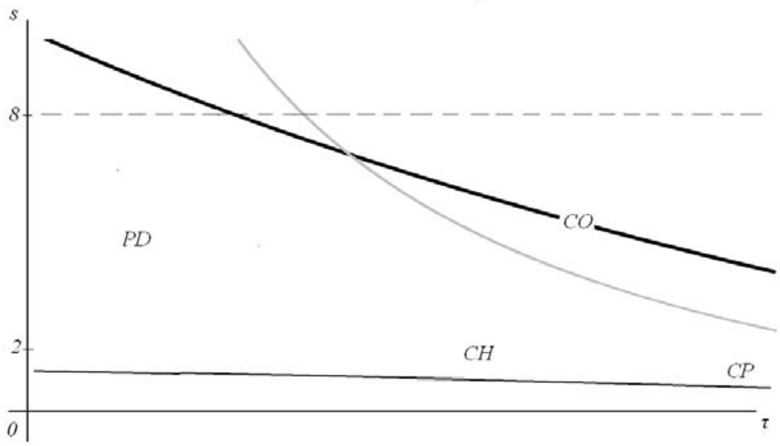

Figure 1: Figure 2: Nature of the Game in Table 1.

is the graph of the function l and thick curve is the graphs of the function h. The figure indicate the regions of parameters for which the game of Table 1 is a member of the different classes in the taxonomy described above. A *PD* game arises for low tax rates if $s > 1.385$. For high tax rates (i.e., for value of τ near $\frac{1}{2}$) a *CP* game arises when the Latin market is not too small relative to the Greek market (i.e., when s is near 1), and a *CO* arises when the Latin market is significantly smaller than the Greek market. For intermediate tax rates *CH* game arises when the Latin market is not too small relative to the Greek market, and a *CO* arises when the Latin market is significantly smaller than the Greek market.

We summarize these results in Proposition 5 below.

Proposition 5: *Assume that $\tau < 1/2$ and $\Delta = 0$. Depending on the values of τ and s the game facing parents when they choose their accounting policies may be a Coordination Game, a Cooperation Game, a Game of Chicken or a Prisoners' Dilemma Game. In particular for most of the size difference between markets, when the tax rates are high, there is an equilibrium in which parents keep one set of books (this equilibrium is unique when the both markets are similar in size) and when tax rates are low, keeping two sets of books is the unique equilibrium. Asymmetric accounting policies, where one parent keeps one set of books and the other keeps two sets of books,* *may be sustained in equilibrium when the size of the Latin market is not too small relative to that of the Greek market.*

Propositions 5 provides a rationale for the mixed empirical evidence on the use of alternative accounting system. Also it identifies the parameter constellations for which there are strategic incentives for maintaining one set of books, i.e., for using the same transfer prices for tax reporting and for managerial purposes. Since keeping one set of books provides parents with an instrument to soften competition in the Greek market, our analysis provides a convincing explanation of how the choice of the accounting policy can serve as a precommitment device. In our setting, the regulatory constraint (i.e., transfer prices for tax purposes must be consistent with the ALP) introduces possibilities for tacit coordination, and provides a rationale for why parents delegate quantity decisions to subsidiaries.

7 Summary and Discussion

The OECD recommendation that transfer prices between parent firms and their subsidiaries be consistent with ALP for tax purposes does not restrict internal pricing policies. Since transfer prices serve both to allocate costs to subsidiaries and to determine tax liability in the jurisdictions where firms operate, the incentive and tax transfer prices would be different. When firms use the same transfer price (and hence, a transfer price consistent with the ALP) for tax reporting and for providing incentives, it is said that they keep one set of books, and when firms use different transfer prices for each purpose, it is said that they keep two sets of books. Thus, in practice, the adoption of ALP commits the firms to the adoption of one of these accounting policies.

In a context of imperfectly competitive markets where firms are vertical separated, we find that accounting policies determine the properties of market outcomes: if parents keep one set of books, competition in the external (home) market softens (intensifies) relative to the equilibrium where parents and subsidiaries are integrated. In contrast, if firms keep two sets of books or keep asymmetric accounting policies (i.e., one parent choosing one set of books and other parent choosing two sets of books), competition intensifies in both markets.

In this paper we show that the choice between one and two sets of books may serve as a precommitment device. When parents choose their accounting policies there exists a wide variety of game forms for alternative parameter depending on the differences in the market size and tax rates. The possible types of the game varies from a prisoners' dilemma (with a unique Pareto inefficient Nash equilibrium in which both parents choose two sets of books) to a game of chicken (with one parent choosing one set of books and other parent choosing two sets of books) or a coordination game (in which both parents choose two sets of books or both parents choose one set of books). Also, parameter constellations of market sizes and tax rates can be found such the type of game is a cooperation game (with a unique Pareto efficient Nash equilibrium in which both parents choose one set of books).

This result provides a possible explanation for the mixed empirical evidence on the use of alternative accounting system. Particularly, the choice of a Pareto superior strategy (i.e., one set of books) can be supported as an equilibrium action under broad conditions. Specifically, for most of the size difference between markets, when the tax rates are high, there is an equilibrium in which parents keep one set of books. Interestingly, the prospect to tacit coordination may contribute to a better understanding of why firms decentralize. Therefore, vertical separation of parent and subsidiary firms, whose motivation is not well understood in the absence of frictions when quantities are strategic substitutes, may be justified if firms keep one set of books.

Appendix 1

If $\tau < \frac{1}{2}$, *then* $\frac{\partial q^T}{\partial \Delta} < 0$ *for all* s.

Proof. We have

$$\frac{\partial q^T}{\partial \Delta} = -\frac{(1-\tau)}{\beta \left(15 (1-\tau)(\Delta - (1-\tau)) + 2s\Delta (\Delta - \tau)\right)^2} \Psi,$$

where

$$\Psi = 5(3-7\tau) + 10\left(\Delta(\Delta - 2(1-\tau)) + 2\tau^2\right) + 2s(\Delta-\tau)^2.$$

Write $\overline{\mu}(\Delta, s)$ and $\overline{\overline{\mu}}(\Delta, s)$ for the value τ that solves $\Psi = 0$ given Δ and s. We omit the expressions of $\overline{\mu}(\Delta, s)$ and $\overline{\overline{\mu}}(\Delta, s)$ because of its length. Then we have $\Psi > 0$, whenever $\overline{\mu}(\Delta, s) < \tau < \overline{\overline{\mu}}(\Delta, s)$ and $\Psi < 0$, otherwise. Since $\tau = \frac{1}{2}$ is the minimum value of $\overline{\mu}(\Delta, s)$ which is yield when $\Delta = \frac{1}{2}$ for all s, then $\tau < \frac{1}{2}$ implies $\tau < \overline{\mu}(\Delta, s)$, and therefore $\Psi > 0$. Thus if $\tau < \frac{1}{2}$, then $\Psi > 0$ and therefore $\frac{\partial q^T}{\partial \Delta} < 0$ for all s. $\square$

Appendix 2

If Δ *is sufficiently low, there is a critical value* $\overline{s}$ *such that* $\frac{\partial \chi^T}{\partial \Delta} \lesseqgtr 0$ *whenever* $s \gtreqless \overline{s}$.

Proof. We have

$$\frac{\partial \chi^T}{\partial \Delta} = \frac{2(1-\tau)}{\beta \left(2s\Delta(\Delta-\tau) - 15(1-\tau)(1-(\tau+\Delta))\right)^2} \overline{\Psi},$$

where

$$\overline{\Psi} = 15(1-\tau)^2 - 2s\left(2\Delta^2 - 3(1-\tau)(2\Delta - \tau)\right).$$

Write $\mu(\Delta, s)$ for the value of τ that solve $\overline{\Psi} = 0$ given Δ and s. We omit the expression of $\mu(\Delta, s)$ because of its length. Then we have $\overline{\Psi} \gtreqless 0$, and therefore $\frac{\partial \chi^T}{\partial \Delta} \gtreqless 0$, whenever $\tau \lesseqgtr \mu(\Delta, s)$. In the limit, as s approaches zero, $\mu = 1$ for all Δ and as s approaches infinity, $\mu = 0$ if $\Delta = 0$. Note that $\tau \in (0, 1)$ and μ decreases with s for all Δ. Since $\lim_{s \to 0} \mu(\Delta, s) = 1$ and $\lim_{s \to \infty} \mu(0, s) = 0$, then μ decreases with s for all Δ implies there is a critical value $\overline{s}$ such that $\tau \lesseqgtr \mu(0)$ whenever $s \gtreqless \overline{s}$, and therefore $\overline{\Psi} \lesseqgtr 0$. Thus we have $\overline{\Psi} \lesseqgtr 0$, and therefore $\frac{\partial \chi^T}{\partial \Delta} \lesseqgtr 0$, whenever $s \gtreqless \overline{s}$. $\square$

Appendix 3

If $l(\tau) < s < 8$, *then* $\Pi_2^* - \Pi_1^* > 0$.

Proof. Assume $\tau < 1/2$, $\Delta = 0$ and $l(\tau) < s < 8$. If $s < h(\tau)$, we calculate the difference of profits between parent 2 and parent 1 at equilibrium described in (4.1) of Proposition 4 as

$$\Pi_2^* - \Pi_1^* = \frac{3\left(1-\tau\right)}{br(\tau,s)^2}\left(s\left(1-\tau\right)\left(43-35\tau+3s\left(3-7\tau\right)\right)-s^3\tau\left(1-2\tau\right)-48\left(1-\tau\right)^2\right).$$

We omit the expression of $\bar{\gamma}(\tau)$ and $\bar{\bar{\gamma}}(\tau)$ for the values of s that solves $\Pi_2^* - \Pi_1^* = 0$ for $\tau < \frac{1}{2}$ because of its length. We have $\Pi_2^* - \Pi_1^* > 0$ whenever $\bar{\gamma}(\tau) < s < \bar{\bar{\gamma}}(\tau)$. Since $\bar{\gamma}(\tau) < l(\tau) < s < \min\{h(\tau), 8\} < \bar{\bar{\gamma}}(\tau)$, then $\Pi_2^* - \Pi_1^* > 0$. If $s > h(\tau)$, we calculate the difference of profits between parent 2 and parent 1 at equilibrium described in (4.2) of Proposition 4 as $\Pi_2^* - \Pi_1^* = \frac{9}{16}\Pi_G^C > 0$. Therefore if $l(\tau) < s < 8$, then whether $s < h(\tau)$, or $s \geq h(\tau)$, $\Pi_2^* - \Pi_1^* > 0$. $\square$

Appendix 4

Let us study the total surplus in a situation where parents use asymmetric accounting policies in term of the total surplus when both firms use two sets of books:

If $h(\tau) \leq s < 8$ *and* $\gamma(\tau) < s$, *then* $S_L^* + S_G^* < S^T$.

Proof. Assume $\tau < 1/2$, $\Delta = 0$, $h(\tau) \leq s < 8$ and $\gamma(\tau) < s$. Using equations (6) and (18) we calculate the total surplus at equilibrium described in (4.2) of Proposition 4 as

$$S_L^* + S_G^* = S^T - \frac{9}{800\beta},$$

and therefore

$$S_L^* + S_G^* < S^T.\square$$

If $\gamma(\tau) > s > h(\tau)$, *then* $S_L^* + S_G^* < S^T$.

Proof. Assume $\tau < 1/2$, $\Delta = 0$ and $\gamma(\tau) > s > h(\tau)$. Using equations (6) and (17) we calculate the total surplus at equilibrium described in (4.2) of Proposition 4 as

$$S_L^* + S_G^* = S^T + \frac{S}{7200b\left(1-\tau\right)^2},$$

where

$$S = 64s^2\tau^2 + 400\left(1-\tau\right)^2 - s\left(239\tau+81\right)\left(1-\tau\right).$$

We omit the expression of $\omega(\tau)$ for the value of s that solves $S = 0$ given τ because of its length. Then we have $S \gtreqless 0$, and therefore $S^{OT} \gtreqless S^T$, whenever $s \gtreqless \omega(\tau)$. Since

$$\omega(\tau) - \gamma(\tau) < 0,$$

for all τ, then $\omega(\tau) - \gamma(\tau) < 0$ implies

$$s < \omega(\tau),$$

and therefore

$$S_L^* + S_G^* < S^T.\square$$

If $\gamma(\tau) < s < h(\tau)$ *and* $s < 8$*, then* $S_L^* + S_G^* < S^T$*.*

Proof. Assume that $\tau < 1/2$, $\Delta = 0$, $\gamma(\tau) < s < h(\tau)$ and $s < 8$. Using equations (6) and (18) we calculate the total surplus at equilibrium described in (4.1) of Proposition 4 as

$$S_L^* + S_G^* = S^T - \frac{\overline{S}}{50br(\tau, s)^2},$$

where

$$\overline{S} = s\left(3(283 - 683\tau)(1-\tau) - 3s^2\tau(10 - 17\tau) + s(235 + \tau(589\tau - 724))\right) + 3600(1-\tau)^2.$$

We omit the expression of $\overline{\omega}(\tau)$ for the value of s that solves $\overline{S} = 0$ given τ because of its length. Then we have $\overline{S} \gtreqless 0$, and therefore $S^{OT} \gtreqless S^T$, whenever $s \gtreqless \overline{\omega}(\tau)$. Since

$$\overline{\omega}(\tau) - h(\tau) < 0,$$

for all τ, then $\overline{\omega}(\tau) - h(\tau) < 0$ implies

$$s < \overline{\omega}(\tau),$$

and therefore

$$S_L^* + S_G^* < S^T.\square$$

If $s < h(\tau)$*,* $s < \gamma(\tau)$ *and* $l(\tau) < s < 8$*, then* $S_L^* + S_G^* < S^T$*.*

Proof. Assume that $\tau < 1/2$, $\Delta = 0$, $s < h(\tau)$, $s < \gamma(\tau)$ and $l(\tau) < s < 8$. Using equations (6) and (17) we calculate the total surplus at equilibrium described in (4.1) of Proposition 4 as

$$S_L^* + S_G^* = S^T + \frac{\widetilde{S}}{450b(1-\tau)^2 r(\tau, s)^2},$$

where

$$\widetilde{S} = 4s^4\tau^2 (5-7\tau)^2 - 18\,000 (1-\tau)^4 - s^3\tau (1-\tau)(230 - \tau (1631 - 1865\tau))$$
$$-2s^2 (745 - \tau (2474\tau - 17))(1-\tau)^2 - 3s (493\tau + 547)(1-\tau)^3 .$$

Since $\widetilde{S}$ is negative for all s if $\tau = \frac{1}{2}$, then $\widetilde{S}$ increases with τ (recall $\frac{\partial q^T}{\partial \tau} > 0$ and $\frac{\partial \chi^T}{\partial \tau} = 0$) implies $\widetilde{S}$ is negative in the space (τ, s), and therefore

$$S_L^* + S_G^* < S^T.\square$$

Appendix 5

Let us study total profits of the firm using one set of books in a situation where parents use asymmetric accounting policies in term of total profits if both parents use two sets of books:

If $h(\tau) \leq s < 8$ *and* $\gamma(\tau) < s$*, then* $\Pi^{\overline{O}T} < \Pi^T$*.*

Proof. Assume $\tau < 1/2$, $\Delta = 0$, $h(\tau) \leq s < 8$ and $\gamma(\tau) < s$. Using (16), we calculate firm's total profits under one set of books when its competitor keeps two sets of books described in (4.2) of Proposition 4 as

$$\Pi^{\overline{O}T} = \Pi^T - \frac{7(1-\tau)}{400\beta},$$

and therefore

$$\Pi^{\overline{O}T} < \Pi^T.\square$$

If $\gamma(\tau) > s > h(\tau)$*, then* $\Pi^{\overline{O}T} < \Pi^T$*.*

Proof. Assume $\tau < 1/2$, $\Delta = 0$ and $\gamma(\tau) > s > h(\tau)$. Using (15), these profits can be calculated as

$$\Pi^{\overline{O}T} = \Pi^T + \frac{\Pi}{3600b(1-\tau)},$$

where $\Pi = 32s^2\tau^2 - s(63 - 143\tau)(1-\tau) - 400(1-\tau)^2$. We omit the expression of $\phi(\tau)$ for the value of s that solves $\Pi = 0$ given τ because of its length. Then we have $\Pi \gtreqless 0$, and therefore $\Pi^{\overline{O}T} \gtreqless \Pi^T$, whenever $s \gtreqless \phi(\tau)$. Since

$$h(\tau) - \phi(\tau) < 0 \text{ and } \gamma(\tau) - \phi(\tau) < 0,$$

for all τ, then $h(\tau) - \phi(\tau) < 0$ and $\gamma(\tau) - \phi(\tau) < 0$ implies

$$s < \phi(\tau),$$

and therefore

$$\Pi^{\overline{O}T} < \Pi^{T}.\square$$

If $\gamma(\tau) < s < h(\tau)$ *and* $s < 8$, *then* $\Pi^{\overline{O}T} < \Pi^{T}$.

Proof. Assume that $\tau < 1/2$, $\Delta = 0$, $\gamma(\tau) < s < h(\tau)$ and $s < 8$. Using (16), we calculate firm's total profits under one set of books when its competitor keeps two sets of books described in (4.1) of Proposition 4 as

$$\Pi^{\overline{O}T} = \Pi^{T} - \frac{3(1-\tau)}{25br(\tau,s)^2}\overline{\Pi},$$

where

$$\overline{\Pi} = (1-\tau)\left(s(109+91\tau) + s^2(85-149\tau)\right) - s^3\tau(5-16\tau) - 1200(1-\tau)^2.$$

Write $\overline{\phi}_1(\tau)$ for the value of s that solves $\overline{\Pi} = 0$ if $\tau < \frac{5}{16}$ and $\overline{\phi}_2(\tau)$ for the value of s that solves $\overline{\Pi} = 0$ if $\tau \notin \left(0, \frac{5}{16}\right)$ (i.e., there are two real roots for the value of s that solves $\overline{\Pi} = 0$: one in the interval $\tau \in \left(0, \frac{5}{16}\right)$ and the other $\tau \notin \left(0, \frac{5}{16}\right)$). We omit the expressions of $\overline{\phi}_1(\tau)$ and $\overline{\phi}_2(\tau)$ for the value of s that solves $\overline{\Pi} = 0$ given τ because of its length.

We have $\overline{\Pi} \gtreqless 0$, and therefore $\Pi^{\overline{O}T} \lesseqgtr \Pi^{T}$, whenever $s \gtreqless \overline{\phi}_1(\tau)$. Since

$$\gamma(\tau) - \overline{\phi}_1(\tau) > 0,$$

for $\tau < \frac{5}{16}$, then $\gamma(\tau) - \phi_1(\tau) > 0$ implies

$$s > \overline{\phi}_1(\tau),$$

and therefore $\Pi^{\overline{O}T} < \Pi^{T}$. We have $\overline{\Pi} \gtreqless 0$, and therefore $\Pi^{\overline{O}T} \lesseqgtr \Pi^{T}$, whenever $s \gtreqless \phi_2(\tau)$. Since

$$\gamma(\tau) - \overline{\phi}_2(\tau) > 0,$$

for $\tau > \frac{5}{16}$, then $\gamma(\tau) - \overline{\phi}_2(\tau) > 0$ implies

$$s > \overline{\phi}_2(\tau),$$

and therefore $\Pi^{\overline{O}T} < \Pi^{T}$. Thus whether $\tau \in \left(0, \frac{5}{16}\right)$, or $\tau \notin \left(0, \frac{5}{16}\right)$,

$$\Pi_2^* - \Pi_1^* > 0.\square$$

If $s < h(\tau)$, $s < \gamma(\tau)$ *and* $l(\tau) < s < 8$, *then* $\Pi^{\overline{O}T} < \Pi^T$ *whenever* $s < 1.385$, *whereas* $\Pi^{\overline{O}T} \lesseqgtr \Pi^T$ *whenever* $\tau \lesseqgtr \widehat{\tau}$ *and* $s > 1.385$.

Proof. Assume that $\tau < 1/2$, $\Delta = 0$, $s < h(\tau)$, $s < \gamma(\tau)$ and $l(\tau) < s < 8$. Using (15), these profits can be calculated as

$$\Pi^{\overline{O}T} = \Pi^T + \frac{\widehat{\Pi}}{225b(1-\tau)\, r(\tau,s)^2},$$

where

$$\begin{aligned}\widehat{\Pi} = &-(1-\tau)^2\left(3s(2981 - 2941\tau)(1-\tau) + 4s^2\left(730 - 2317\tau + 1444\tau^2\right)\right) \\ &-s^3\tau(1-\tau)\left(437\tau - 5\tau^2 - 260\right) + 18\,000(1-\tau)^4 + 2s^4\tau^2(5-7\tau)^2.\end{aligned}$$

There is no closed form solutions for the value of s that solves $\widehat{\Pi}(\tau,s) = 0$. Figure 3 below are the graphs of the function $\widehat{\Pi}$ for different values of τ. As graphically displayed by the Figure 3 if $\tau = \frac{1}{2}$, the values of s must lie between 1 and $\frac{5}{2}$ and $\widehat{\Pi}$ is positive for all s and if $\tau = 0$, the values of s must lie between $\frac{3}{2}$ and 8 and $\widehat{\Pi}$ is negative for all s. Also $\widehat{\Pi}$ increases with τ if $s > 1.385$ and decreases with τ, otherwise. Therefore for $s > 1.385$, since $\widehat{\Pi} < 0$ if $\tau = 0$ and $\widehat{\Pi} > 0$ if $\tau = \frac{1}{2}$, then $\widehat{\Pi}$ increases with τ implies there is a critical value $\widehat{\tau}$ such that $\widehat{\Pi} \lesseqgtr 0$ whenever $\tau \lesseqgtr \widehat{\tau}$. Then we have $\widehat{\Pi} \lesseqgtr 0$, and therefore $\Pi^{\overline{O}T} \lesseqgtr \Pi^T$, whenever $\tau \lesseqgtr \widehat{\tau}$. For $s < 1.385$, since $\widehat{\Pi} > 0$ for $\tau = \frac{1}{2}$, then $\widehat{\Pi}$ decreases with τ implies $\widehat{\Pi} > 0$ for all τ and therefore $\Pi^{\overline{O}T} > \Pi^T$.□

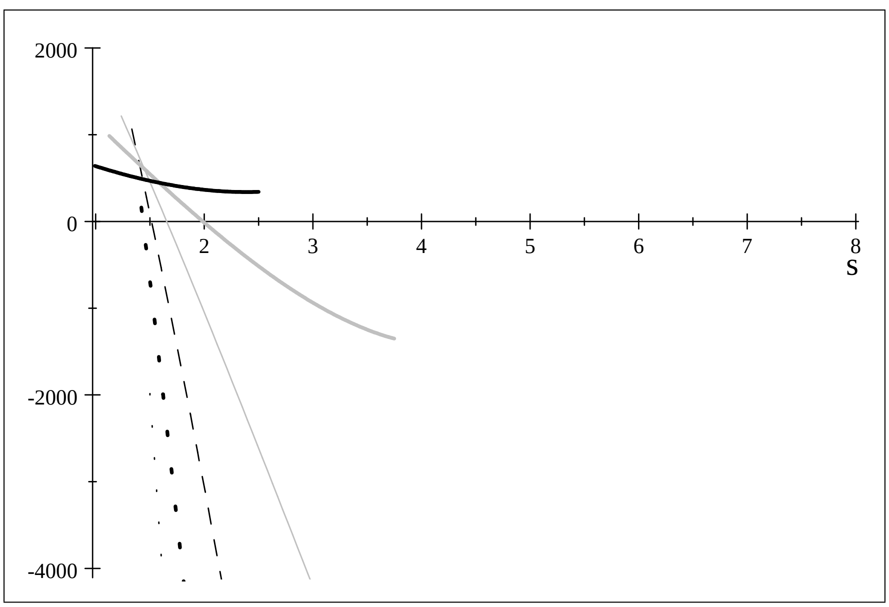

Figure 3: calculating $\widehat{\Pi}$ for different values of τ and s.

Appendix 6

Let us study the profits of the firm using two sets of books in a situation where parents use asymmetric accounting policies in term of total profits if both parents use one set of books:

If $h(\tau) \leq s < 8$, *then* $\Pi^{\overline{O}T} < \Pi^{T}$.

Proof. Assume $\tau < 1/2$, $\Delta = 0$ and $h(\tau) \leq s < 8$. Using (12), we calculate firm's total profits under two sets of books when its competitor keeps one set of books described in (4.2) of Proposition 4 as

$$\Pi^{O\overline{T}} = \Pi^{O} - \frac{3\left(1-\tau\right)}{8b\left(8s+27\right)^{2}}\left(216+117s+16s^{2}\right),$$

and therefore

$$\Pi^{O\overline{T}} < \Pi^{O}.\square$$

If $l\left(\tau\right) < s < h(\tau)$ *and* $s < 8$, *then* $\Pi^{O\overline{T}} > \Pi^{O}$ *whenever* $s \in (1.23, 2.26)$, *whereas* $\Pi^{O\overline{T}} \lesseqgtr \Pi^{O}$ *whenever* $\tau \gtreqless \widetilde{\tau}$ *and* $s \notin (1.23, 2.26)$.

Proof. Assume that $\tau < 1/2$, $\Delta = 0$, $l\left(\tau\right) < s < h(\tau)$ and $s < 8$. Using (12), we calculate firm's total profits under two sets of books when its competitor keeps one set of books described in (4.1) of Proposition 4 as

$$\Pi^{O\overline{T}} = \Pi^{O} + \frac{3\left(1-\tau\right)}{b\left(8s+27\right)^{2}r(\tau,s)^{2}}\widetilde{\Pi},$$

where

$$\begin{aligned}\widetilde{\Pi} &= 3s^{3}\left(555-\tau\left(2222-1769\tau\right)\right)-\left(8s^{5}\left(1+\tau\right)\left(3-5\tau\right)+4s^{4}\left(21+\tau\left(130-203\tau\right)\right)\right)\\ &\quad-\left(15\,552\left(1-\tau\right)^{2}-9\left(s^{2}\left(1009-\tau\left(2450-1429\tau\right)\right)+18s\left(1-\tau\right)\left(43-27\tau\right)\right)\right).\end{aligned}$$

There is no closed form solutions for the value of s that solves $\widetilde{\Pi} = 0$. Figure 4 below are the graphs of the function $\widetilde{\Pi}$ for different values of τ. As graphically displayed by the Figure 4 if $\tau = \frac{1}{2}$ the values of s must lie between 1 and 4 and $\widetilde{\Pi}$ is positive for all $s \in (1.23, 2.26)$ and if $\tau = 0$, the values of s must lie between $\frac{3}{2}$ and 8 and $\widetilde{\Pi}$ is positive for all s. Also $\widetilde{\Pi}$ decreases with τ for all s. For $s \in (1.23, 2.26)$, since $\widetilde{\Pi} > 0$ for $\tau = \frac{1}{2}$, then $\widetilde{\Pi}$ decreases with τ implies $\widetilde{\Pi} > 0$ for all τ and therefore $\Pi^{O\overline{T}} > \Pi^{O}$.

For a given $s \notin (1.23, 2.26)$, since $\widetilde{\Pi} < 0$ for $\tau = \frac{1}{2}$ and $\widetilde{\Pi} > 0$ for $\tau = 0$, then $\widetilde{\Pi}$ decreases with τ implies there is a critical value $\widetilde{\tau}$ such that $\widetilde{\Pi} \lesseqgtr 0$ whenever $\tau \gtreqless \widetilde{\tau}$. Then we have $\widetilde{\Pi} \lesseqgtr 0$, and therefore $\Pi^{O\overline{T}} \lesseqgtr \Pi^{O}$, whenever $\tau \gtreqless \widetilde{\tau}$.□

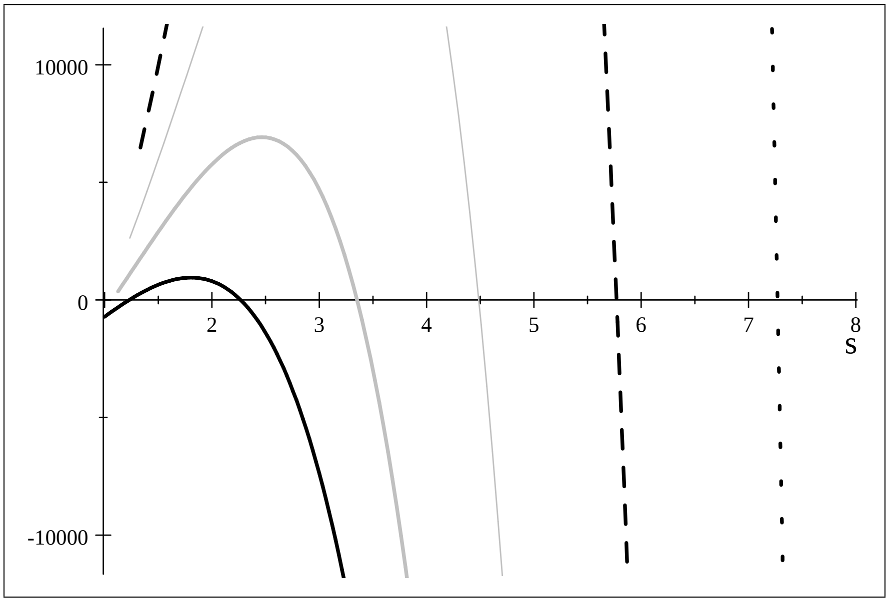

Figure 4. Graphs of the function $\widetilde{\Pi}\,(s)$ for different values of τ: 0 (thin dotted curve); 0.1 (thick dotted curve); 0.2 (dashed curve); 0.3 (thin gray curve); 0.4 (thick gray curve); and 0.5 (thick black curve).

If $\gamma(\tau) < s < h(\tau)$, *then* $\Pi^{O\overline{T}} < \Pi^{O}$.

Proof. Assume that $\tau < 1/2$, $\Delta = 0$, $\gamma(\tau) < s < h(\tau)$. Since the sign of $\Pi^{\overline{O}T} - \Pi^{T}$ is positive, we discuss the sign of $\Pi^{O} - \Pi^{O\overline{T}}$ in order to characterize the SPE in this region of parameters. For $s = \gamma(\tau)$, the equation above yields $\widetilde{\Pi} < 0$ for all τ. Since $\widetilde{\Pi} < 0$ if $s = \gamma(\tau)$, then $\widetilde{\Pi}$ decreases with τ implies $\widetilde{\Pi} < 0$ for all $s > \gamma(\tau)$ and therefore $\Pi^{O\overline{T}} < \Pi^{O}$.□

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